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Shrinking a Wet Deposition Network

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Abstract

We use the spatial-temporal model described in Oehlert (1993) to choose good sets of candidate stations for deletion from the NADP/NTN network. We use three criteria: that the sum of 11 regional trend estimate variances be as small as possible, that the sum of local trend estimation variance be as small as possible, and that the sum of local mean estimation variance be as small as possible. Good choices of stations for deletion result in a modest increase in criteria (about 7% to 12%) for 100 stations deleted from the network, while random sets of 100 stations can increase criteria by a factor of 2 or more.

1 Introduction

Oehlert (1993) presented a model for wet deposition data in North America that allows the estimation of spatial means and trends. Oehlert (1994) used this model for several purposes, including determining trend detectability and quantifiability and network modification via the deletion or addition of 10 stations. This note extends the station deletion results of Oehlert (1994) to the deletion of 100 stations rather than just 10. Refer to Oehlert (1993) for the details of the statistical model; a short description is given in the Appendix.

As budgets decrease, there is pressure to shrink the networks that monitor acid deposition by deleting stations from the networks. At the end of 1987, there were 249 monitoring stations operating in the NADP/NTN (USA) and CANSAP and APIOS-C (Canada) networks. The locations

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of these stations are shown in Figure 1. One question is "How can we shrink these networks while retaining as much information about regional mean and trend as possible?" We consider how much information on mean or trend is lost when 100 of the 195 NADP/NTN stations are deleted.

We measure information using three criteria. The spatial trend model tiles part of North American with 798 rectangles of size 1^o latitude by 1.5^o longitude; 616 of these rectangles touch the USA, the remainder touch Canada. The model produces mean and trend estimates with corresponding estimation variances for each of these rectangles. The first information criterion is simply the sum of these 616 rectangle trend estimate variances, or equivalently, the average variance of trend estimation at the local level. We divide the 616 rectangles into 11 regions as shown in Figure 2 and compute the mean trend for each region. Our second criterion is the sum of the variances of these 11 regional averages. The first criterion will tend to keep a fairly even density of stations and estimate the smaller regions poorly, while the second criterion will keep stations in or near the smaller regions at the expense of the larger regions. The third criterion is the sum of the 616 rectangle mean estimate variances. This criterion will also tend to keep stations uniformly.

2 Variance parameter and station selection

Estimation variance depends on the length of the data series under consideration and the shape of the trend, the error structure in the data, the station configuration (number and locations), and the spatial smoothing parameters. We use a 25 year data series with assumed underlying trend having shape as shown in Figure 3 (R. Dennis, personal communication). The estimated parameters are the mean deposition and total change in wet deposition between 1985 and 2010 (though all change actually occurs between 1990 and 2005). Thus our station selection is hypothetically taking place in the future, based on 25 years of data. As we will be using data on the log scale, trend coefficients (and standard errors) can be interpreted roughly as percent change/100, and the standard error for the mean is roughly the coefficient of variation for the estimate.

The variance structure of the data is determined by the S, D, C, and σ_δ parameters described in the Appendix. From these parameters and the shape of the trend, we may compute the covariance matrix of the estimated station means or trends (Equations 5 and 7 in the Appendix). The matrix S is a spatial covariance, and we use an exponential spatial covariance (Kriging type) fit to log sulfate wet deposition data from all stations having no missing data from 1982-1986, as in Oehlert (1993). For trend, the term in Equation 5 involving S and C is a matrix with 0.00361 on the diagonal, and off diagonal elements equal to $0.000479\exp(-0.00214d)$, where d is the distance between two stations measured in km, and the term involving D is a constant matrix with values 0.000191. The corresponding matrix involving S and C for station means has 0.00728 on the diagonal and $0.000966\exp(-0.00214d)$ on the off diagonals, where again d is the distance between two stations measured in km, and the term involving D for means is a constant matrix with values 0.001073. The individual station offset variance σ_δ^2 is 0.0201. Thus the trends are better determined than the means. Also note that in our model the variance of a station mean will never be less than σ_δ^2, which does not decrease as additional data accrues. Since the continental scale variance D appears as a constant added to the covariance matrix of the rectangle values, changing the size of the component due to D changes the criteria by a fixed amount and does not alter the selection of stations for deletion.

We may view the spatial smoothing parameter (λ_α for means or λ_β for trend) from a Bayesian perspective as (approximately) a prior precision on the difference in values (means or trend) between adjacent rectangles. We use two values for λ_β (100 and 25) implying that neighboring rectangles should have the same trend value, give or take 0.1 or 0.2 respectively, and two values for λ_α (25 and 1) corresponding to neighboring rectangles having the same log mean, give or take 0.2 or 1. These represent very weak smoothing, particularly for the trends; larger values of the smoothing parameter will lead to less variance inflation when stations are deleted.

Station selection takes place sequentially using the so-called greedy algorithm. That is, we first find the station that gives the smallest increase in criterion when deleted. Then, given that the first selected station is deleted, we find the second station as the one that gives the smallest increase in criterion, and so on until we reach the desired number of stations. This sequential selection will not in general find truly optimal sets of stations, but should find good sets of stations. Sequential selection is expensive computationally, and truly optimal selection is much more expensive still. We will refer to our sequentially selected station sets as "greedy".

3 Results for trend

Our results comprise regional trend estimate variances and total rectangle variances for a variety of station sets and smoothing parameters. We used two values of λ_β, 100 and 25. Several sets of stations were evaluated for regional trend variance and total rectangle variance using both parameters. The sets of stations include:

- all stations,
- 100 stations deleted in a greedy manner when the deletion criterion is total regional variance (deletions done separately for both smoothing parameters),
- 100 stations deleted in a greedy manner when the deletion criterion is total rectangle variance (deletions done separately for both smoothing parameters),
- 100 sets of stations with 100 randomly chosen stations deleted in each,
- all USA stations deleted.

Before addressing specific criterion values, let us examine general patterns of stations selected. First, presence or absence of the continental effect changes the value of the criterion observed, but does not change the stations selected for deletion. Next, changing the spatial smoothing parameter from 100 to 25 does change the stations selected; when using regional variances as a criterion, 10 of the 100 stations changed between values of λ_β, and when using total rectangle variance as a criterion, 7 of the 100 stations changed. Most, but not all, unique stations from a $\lambda_\beta = 10$ list were geographically close to unique stations in the $\lambda_\beta = 5$ list.

Finally, the station lists selected for deletion using regional or rectangle criteria generally had about 70 stations in common, but gave very different spatial appearances. Figures 4 and 5 show the deleted and kept stations for the regional and rectangle criteria when the smoothing parameter is 100. The regional criterion deletes many more stations from the plains, while the rectangle criterion deletes many more stations from the East. The stations remaining show different patterns also. The rectangle criterion keeps a reasonably uniform set of stations, while the regional criterion prefers to keep stations in and near the smaller regions.

Tables 1 and 2 show the regional and rectangle criterion results when the smoothing parameters are 100 and 25 respectively. If the continental component is set to zero, regional trend variances are smaller by 1.91×10^{-4}, the regional total variance is smaller by 20.97×10^{-4}, and the rectangle total variance is smaller by 0.117 for all station configurations.

Look first to the total regional variance for the smoothing parameter of 10. The total when using all stations is 0.0091, while that for the greedy stations optimizing on the regional criterion is 0.0098, a 7% increase. The stations selected using the rectangle criterion have total 0.0107, a 17% increase. The randomly chosen station sets produce totals ranging from 25% to 97% more than the

Table 1: Regional trend variances $\times 10000$ and total rectangle variances for station sets evaluated with base error covariance and smoothing parameter 100.

	deletions								
	none	criterion and smoothing				random			all USA
		reg.100	reg.25	rect.100	rect.25	min	mean	max	
Northeast	6.57	7.05	6.94	8.62	8.75	6.69	8.44	11.97	16.40
MidAtlantic	9.51	9.74	9.75	10.92	12.22	10.08	13.64	20.17	41.93
Appalachian	5.05	5.74	5.83	8.50	8.11	5.54	6.35	7.80	19.69
Midwest	4.53	5.82	5.69	6.15	5.93	4.89	5.50	6.36	14.78
South	5.80	6.19	6.39	6.88	6.82	6.77	9.41	21.52	74.14
Plains	3.72	5.34	5.36	3.95	3.94	4.23	4.79	5.87	31.12
Intermtn/desert	5.27	5.97	5.92	5.59	5.71	5.80	7.70	11.89	70.59
Southwest CA	23.86	23.93	24.00	23.89	23.98	23.97	38.55	72.97	135.98
Northwest	10.56	10.77	10.77	12.31	12.31	10.66	18.35	36.99	103.37
Miss. Delta	11.34	11.56	11.56	13.46	12.42	11.69	16.26	27.58	81.25
Rocky Mts.	5.19	6.12	6.11	6.45	6.45	5.69	6.65	7.72	54.06
Total regional	91.41	98.24	98.32	106.71	106.66	114.34	135.64	180.19	643.32
Total rectangle	2.40	2.79	2.80	2.68	2.68	2.85	2.93	3.06	6.33

Table 2: Regional trend variances ×10000 and total rectangle variances for station sets evaluated with base error covariance and smoothing parameter 25.

	deletions								
	none	criterion and smoothing				random			all USA
		reg.100	reg.25	rect.100	rect.25	min	mean	max	
Northeast	10.60	11.81	11.31	16.53	16.94	10.74	16.70	30.13	47.08
MidAtlantic	18.90	19.38	19.39	22.15	26.37	20.22	34.06	62.37	144.37
Appalachian	6.87	8.43	8.70	18.69	16.81	7.79	10.94	16.78	57.39
Midwest	5.97	9.98	9.36	11.14	10.29	7.08	9.21	12.41	42.17
South	8.65	9.69	10.23	11.69	11.43	11.43	20.04	66.04	278.08
Plains	5.27	10.46	10.49	5.78	5.77	6.76	8.73	12.56	105.75
Intermtn/desert	8.90	10.74	10.53	9.63	9.84	10.21	16.50	31.08	257.16
Southwest CA	60.34	60.38	60.46	60.35	60.45	60.51	118.91	272.55	520.53
Northwest	18.74	18.95	18.95	23.32	23.32	18.89	45.73	117.37	381.08
Miss. Delta	24.37	24.56	24.55	30.81	26.74	24.83	42.90	92.23	307.38
Rocky Mts.	8.48	10.62	10.53	12.05	12.05	9.71	13.23	17.02	192.37
Total regional	177.09	195.02	194.51	222.13	220.01	250.36	336.94	522.56	2333.35
Total rectangle	7.47	9.26	9.28	8.74	8.73	9.54	9.88	10.40	23.91

all-stations total, with a mean increase of 48%. Thus, appropriate selection of stations for deletion can substantially reduce the penalty incurred.

Results for a smoothing parameter of 25 have variances about a factor of 2 greater than for parameter 100. Furthermore, the percent increase in criterion is greater for the smaller smoothing parameter, since we are in effect averaging over a narrower neighborhood. For stations deleted with the regional criterion, there is a 10% increase, while stations deleted with the rectangle criterion show a 25% increase. Random deletions had increases ranging from 41% to 195%, with a mean of 90%.

Next, examine the regionwise results. The Appalachian, Midwest, Plains, and Rocky Mountain regions show relatively little average increase in variance of the random deletions over the full station set (1.3, 0.97, 1.02, and 1.47 $\times 10^{-4}$ respectively). The Northeast and Intermountain Desert regions show moderate increases (1.87 and 2.43 $\times 10^{-4}$ respectively), and the Mid Atlantic, South, Southwest California, Northwest, and Mississippi Delta regions show large increases (4.13, 3.61, 14.69, 7.79, and 4.92 $\times 10^{-4}$ respectively). In those regions where randomly chosen station sets produce large increases the greedy station variance is close to the full station set variance, while in those regions where randomly chosen station sets produce small increases the greedy station variance moves well up into the distribution of random variances. Thus, the greedy station station set achieves a small total variance by keeping the variance as small as possible in those regions where chance deletion might substantially increase the variance, and letting the variance grow in those regions where deletion only increases variance slightly. In particular, large, internal regions lose relatively more stations than small or border regions. This was evident in Figures 4 and 5.

The total rectangle criterion shows much less variation across station sets. For a smoothing parameter of 100, the increases for randomly deleted stations ranged from 19% to 28%, with a mean of 22%, while the stations selected using the rectangle criterion had an increase of 12% and those selected with the regional criterion had an increase of 17%. Increases for the smoothing parameter 25 were larger both absolutely and relatively, but still considerably smaller than increases seen for the regional criterion. This is because the large rectangle variances occur at the boundary of the estimation area and at rectangles that are distant from any station. As long as we do not delete stations near the boundary or the empty regions, the large rectangle variances remain the same and only the small ones change leading to relatively little change in this criterion.

Compare the regional variances for station sets selected by the rectangle and regional criteria, for example, the columns headed by reg.100 and rect.100 in Table 1. The Northeast, MidAtlantic, Appalachian, Northwest, and Mississippi Delta regions have larger regional variances under the rectangle criterion choice than under the regional variance criterion. Conversely, the Plains and Intermountain/desert regions (both very large geographically) have smaller regional variances under the rectangle criterion. Again, this is an illustration of the rectangle criterion favoring uniformity over good estimation for small regions.

Figure 6 shows the regional and rectangle criteria for smoothing parameter 100 as a function of the number of stations deleted. As would be expected, the curves are convex, so that the second set of 50 stations deleted had a much larger criterion penalty than the first set of 50. From the shape of the curve, we would anticipate the next 50 to lead to an even greater increase.

It might be argued that it is unrealistic to expect the Canadian network to remain at full strength during a drastic reduction of the US network. To evaluate the effect of a smaller Canadian network, we repeated our greedy selection of stations using the regional variance criterion and smoothing parameter 100 after randomly deleting 28 of the Canadian stations. Ninety-seven of the 100 selected stations are the same, and the regional and rectangle variance criteria are about 1% larger than when all Canadian stations are included. Thus, at least for this one reduction of the Canadian network, greedy station sets for deletion depended only very slightly on changes in the Canadian network.

The final columns of Tables 1 and 2 show the criteria if all the USA stations are deleted so that the estimates in the USA are extrapolations from the Canadian data. This obviously extreme situation illustrates the effect of the smoothing prior and just how poor the estimation could get.

4 Results for the mean

Our results for the mean parallel those for trend, but are less extensive in that we only did station deletions using the rectangle total criterion. Thus we evaluate variances using smoothing parameters of 25 and 1 and the following station sets:

- all stations,
- 100 stations deleted in a greedy manner when the deletion criterion is total rectangle variance (deletions done separately for both smoothing parameters),
- 100 sets of stations with 100 randomly chosen stations deleted in each (the same sets as for trend),

• all USA stations deleted.

As with trend, changing the smoothing parameter only slightly changes the stations selected for deletion; 91 of the 100 stations are the same for the two smoothing parameters. Moreover, 93 of the 100 stations chosen for deletion using the smoothing parameter of 25, rectangle total criterion, and the covariance for means also appeared in the list of stations to delete for trend when using the smoothing parameter 100 and the rectangle total criterion. Thus the stations selected for deletion seem to depend more on the optimization criterion (regional versus rectangle total) than on whether we optimize for trend or mean.

Tables 3 and 4 show the regional and rectangle criterion results for means and smoothing parameters 25 and 1 respectively. The proportional increases in variance criteria due to station deletion for means are similar to those found for trends. Since we have used the total rectangle criterion, the selected stations tend to increase the variances for small regions while keeping the large region variances fairly constant. The increase in rectangle total criterion is about 11%.

5 Discussion

These computations provide some qualitative guidance that is intuitive, and some quantification that I find rather surprising. Qualitatively, if the criterion is total rectangle variance, then you want to end up with an approximately uniform distribution of stations, while if the criterion is total regional variance, you prefer to keep stations in and near small regions. In neither case do you delete stations from the boundary (where estimation is more difficult). Quantitatively, these computations suggest that there can be a substantial reduction in the number of monitoring stations without a large increase in evaluation criteria. Random station deletion, however, can lead to large increases in the criteria. Station sets chosen using any criterion were better than random deletion for all criteria.

These computations also have some important limitations that may narrow their applicability. First and foremost, our results are for criteria relating to estimating regional trends or means over a period of 25 years. They do not tell us about the effects of station deletion on prediction of sulfate fields at shorter time scales or smaller geographic areas. Thus, though we found relatively slight criteria increases when deleting 100 stations, these monitoring networks have multiple uses and there are likely to be substantially larger increases in other criteria when this many stations are deleted. Second, our computations were run with a narrow set of variance and smoothing parameters. Future

Table 3: Regional mean variances ×10000 and total rectangle variances for station sets evaluated with base error covariance and smoothing parameter 25.

		deletions					
	none	smoothing in deletion		random			All USA
		rect.25	rect.1	min	mean	max	
Northeast	30.79	45.15	42.51	31.82	39.94	55.12	71.82
MidAtlantic	45.12	54.78	59.86	49.02	63.90	88.81	177.00
Appalachian	23.22	41.25	37.89	26.54	30.15	36.06	86.42
Midwest	20.67	30.24	28.02	22.70	25.65	29.50	63.08
South	28.59	34.95	34.73	34.27	46.93	96.11	303.41
Plains	18.48	19.81	20.11	21.28	23.91	28.82	133.55
Intermtn/desert	25.85	28.11	28.88	29.10	38.39	57.06	298.32
Southwest CA	117.23	117.62	118.58	118.21	177.73	303.01	558.12
Northwest	53.69	64.27	64.29	54.55	90.43	168.85	436.28
Miss. Delta	54.69	60.18	62.09	57.48	77.15	121.07	332.10
Rocky Mts.	24.25	31.55	31.56	27.29	32.02	37.06	230.92
Total regional	442.56	527.93	528.52	560.44	646.21	819.46	2691.02
Total rectangle	10.63	11.78	11.80	12.38	12.74	13.22	26.03

Table 4: Regional mean variances ×10000 and total rectangle variances for station sets evaluated with base error covariance and smoothing parameter 1.

	deletions						
	none	smoothing in deletion		random			All USA
		rect.25	rect.1	min	mean	max	
Northeast	154.28	357.73	296.25	155.69	297.10	628.98	1047.41
MidAtlantic	332.78	398.21	486.33	359.71	701.32	1424.36	3434.39
Appalachian	75.00	390.08	287.70	91.87	169.38	315.89	1276.44
Midwest	61.81	212.43	160.48	86.87	137.90	215.58	931.08
South	115.01	182.42	175.21	178.05	380.26	1507.29	6819.94
Plains	65.71	75.42	78.01	99.78	147.08	239.09	2517.11
Intermtn/desert	138.93	153.96	157.63	166.85	315.44	664.38	6259.67
Southwest CA	1254.98	1255.10	1256.38	1258.22	2714.43	6675.79	12855.90
Northwest	303.85	402.80	402.80	306.64	949.46	2715.05	9307.85
Miss. Delta	453.84	483.62	500.77	460.55	911.07	2178.04	7561.62
Rocky Mts.	123.55	200.06	200.06	149.79	236.18	334.74	4648.82
Total regional	3079.73	4111.83	4001.63	4786.23	6959.61	11652.10	56660.23
Total rectangle	169.15	203.56	202.19	223.74	232.33	245.48	588.01

work will include validation for other variance and smoothing parameters. Next, these results treat all geographic areas as similar. For example, they take no account of complex topography. Several nearby stations may be needed to characterize deposition in mountainous terrain, but the model used here ignores elevational and terrain effects. Similarly, clusters of stations used in research or stations located at important ecological sites were not protected. Finally, our use of the log transformation to stabilize the variance has the side effect that trends and variances in the log data are effectively relative trends and relative variances in the original data, and thus trends and means in regions of low concentration and deposition have relatively more weight than they would have in the original scale. This may or may not be considered an advantage.

Many other authors have investigated optimal network design using a variety of criteria. For example, Fedorov and Mueller (1989) reviewed and described how observation network design developed independently of classical optimal experimental design; they then showed how the two approaches yield the same procedures for a regression model with random parameters. Cambanis (1985) gives a review of the one dimensional work. The goal in these approaches is the accurate estimation of parameters in a statistical model of the observations. Our work is closest in spirit to these estimative models.

Caselton and Zidek (1984) developed a method (used, for example, in Wu and Zidek (1992)) that views the purpose of a network to be the maximization of information about the unmonitored sites on the basis of the monitored sites. Monitoring stations are ranked on the basis of how much the entropy of unmonitored sites would increase if the monitoring station were deleted. This information maximization is somewhat goal neutral, in that it doesn't presuppose that the network data are used for estimation, prediction, or whatever.

Haas (1992) and Trujillo-Ventura and Ellis (1991) consider multi-objection optimization, wherein some combination of several objectives is optimized. These objectives include minimization of prediction error, minimization of cost, improvement of variance estimation, and increased probability of detecting violations. Both of these papers work in the context of Kriging for the observations.

6 Appendix

Here we present a sketch of the statistical model; see Oehlert (1993) for details. Tile Eastern North America with small rectangles, one degree of latitude by 1.5 degree of longitude. Assume that all stations within a rectangle have the same expected trend and mean, but that trend and mean may vary from rectangle to rectangle. Take as regional trend or mean the unweighted average of the estimated rectangle values for each of the rectangles in the regions of interest. The variance of these regional averages is determined from the covariance matrix of the rectangle values.

There are s monitoring stations each with y years of data. Let Y_i be a vector containing the series of annual values at station i. The values could be concentrations, precipitation adjusted concentrations, depositions, etc. Let $j(i)$ indicate the rectangle in which station i occurs. We assume that each series Y_i has the structure

$$Y_i = \alpha_{j(i)}\mathbf{1} + \mathbf{t}\beta_{j(i)} + L + N_i + \delta_i\mathbf{1}, \tag{1}$$

where $\alpha_{j(i)}$ is the mean value in rectangle $j(i)$, $\mathbf{t}$ is the trend shape in Figure 3 (centered to have mean zero), $\beta_{j(i)}$ is the change for rectangle $j(i)$, L is a long-term noise series common to all stations, N_i is a short-term station specific noise series, and δ_i is a station specific random effect accounting biasing effects such as elevation or proximity to point sources of sulfur. Let N denote the vector of all the station specific noise terms, let δ be the vector of station biases, and let $\alpha_J = (\alpha_{j(1)}, \cdots, \alpha_{j(s)})'$ be the vector of rectangle means, with β_J the analogous vector for trend.

Assume that all station specific noise terms have the same temporal correlation structure, and assume that the cross covariance factors into a spatial term and a temporal term, so that the covariance matrix of N is

$$Cov(N) = S \otimes C, \tag{2}$$

where S is the spatial covariance matrix and C is the common temporal correlation matrix. Because of the structure of the short term noise series, C is essentially tridiagonal, with sub- and super-diagonal elements nearly zero, say about 0.01. The covariance matrix of L is denoted by D. The station random effects δ_i are uncorrelated with variance σ_δ^2, and we assume that N, L, and δ are uncorrelated. Thus the covariance of Y is

$$\mathbf{1}_{s\times s} \otimes D + S \otimes C + \sigma_\delta^2 I_{s\times s} \otimes \mathbf{1}_{y\times y}. \tag{3}$$

With this notation, observed station slopes (calculated via ordinary least squares) can be expressed

$$b = \beta_J + \mathbf{1}_{s\times 1} \times (\mathbf{t}'\mathbf{t})^{-1}\mathbf{t}'L + I_{s\times s} \times (\mathbf{t}'\mathbf{t})^{-1}\mathbf{t}'N, \tag{4}$$

with covariance matrix

$$Cov(b) = \mathbf{1}_{s\times s} \times (\mathbf{t}'\mathbf{t})^{-1}\mathbf{t}'\ D\mathbf{t}(\mathbf{t}'\mathbf{t})^{-1} + S \otimes (\mathbf{t}'\mathbf{t})^{-1}\mathbf{t}'\ C\mathbf{t}(\mathbf{t}'\mathbf{t})^{-1}; \tag{5}$$

and the observed station means can be expressed

$$a = \alpha_J + \bar{L} + (I_{s\times s} \otimes \frac{1}{y}\mathbf{1}'_{y\times 1})N + \delta, \tag{6}$$

(where $\bar{L}$ is the mean of the long-term noise series) with covariance matrix

$$Cov(a) = \mathbf{1}_{s\times s} \times \mathbf{1}'\ D\mathbf{1}/y^2 + S \times \mathbf{1}'\ C\mathbf{1}/y^2 + \sigma_\delta^2 I. \tag{7}$$

We now estimate the β_j for all rectangles j. The estimates b_i serve as the observations in a spatial linear model. For example, an observed trend b_i from station i in the $j(i)$th rectangle has expected value $\beta_{j(i)}$. We may express this in matrix form as

$$E(b) = W\beta, \tag{8}$$

where the matrix W is all zero except for ones in each row indicating rectangle.

In addition to the observed trends, we also use a discrete smoothness prior for β, because we believe that β varies slowly in space. We make this belief explicit by using a partially improper normal prior on β with mean zero and inverse covariance matrix $\lambda_\beta A'A$. The matrix A has a row for every pair of adjacent rectangles, and a column for every rectangle. A is all zeros, except that each row has entries of 1 and -1 for the associated rectangle pair coefficients. The smaller the prior variance for these differences $(1/\lambda_\beta)$, the smoother the resulting estimates will be.

If Σ_b is the covariance matrix of b, then the estimate $\hat{\beta}$ can be expressed

$$\hat{\beta} = (W'\Sigma_b^{-1}W + \lambda_\beta A'A)^{-1}W'\Sigma_b^{-1}b \tag{9}$$

with posterior variance matrix

$$Cov(\hat{\beta}) = (W'\Sigma_b^{-1}W + \lambda_\beta A'A)^{-1}. \tag{10}$$

Similar forms are used for estimating α.

We use an ARMA(1,1) type correlation structure for D: $\rho_k = \rho_1\phi^{k-1}$ for lags $k \geq 1$. Oehlert (1993) used the parameters $\rho_1 = 0.3$ and $\phi = 0.95$, based on analysis of historical precipitation records, and these values will also be used in this study. The variance of this process, σ_L^2 is estimated using covariance between stations at great distances, as discussed in Oehlert (1993).

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Figure 1: Location of Wet Deposition Stations

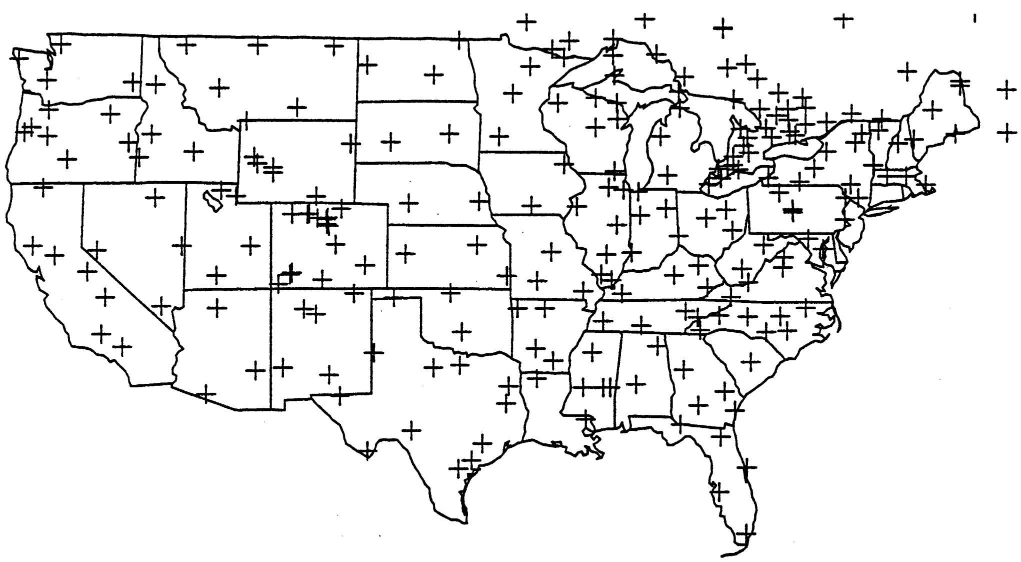
Figure 2: Eleven Regions

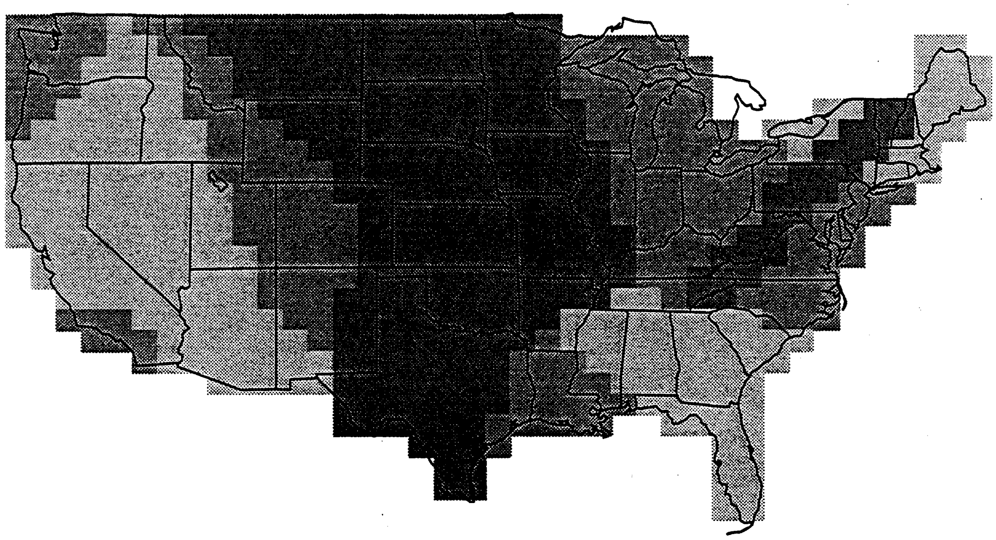
Figure 3: Shape of RADM based reduction

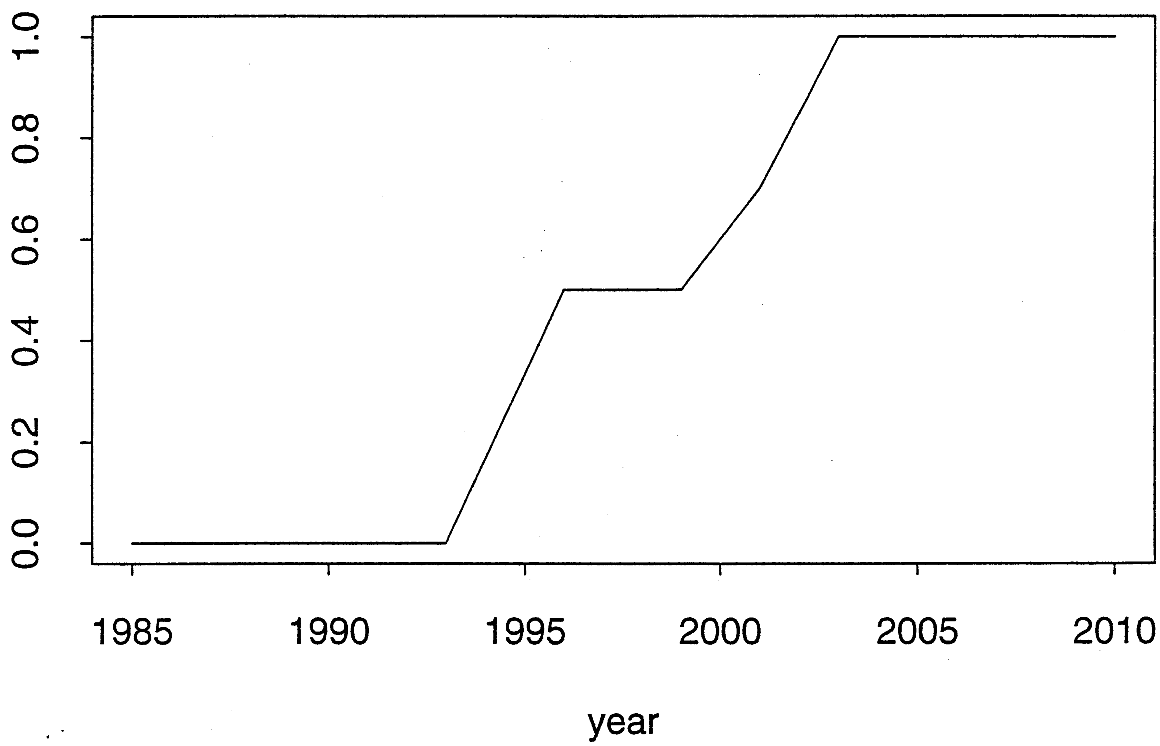
Figure 4: Locations of deleted (X) and not deleted (O) stations using regional criterion and smoothing parameter 10.

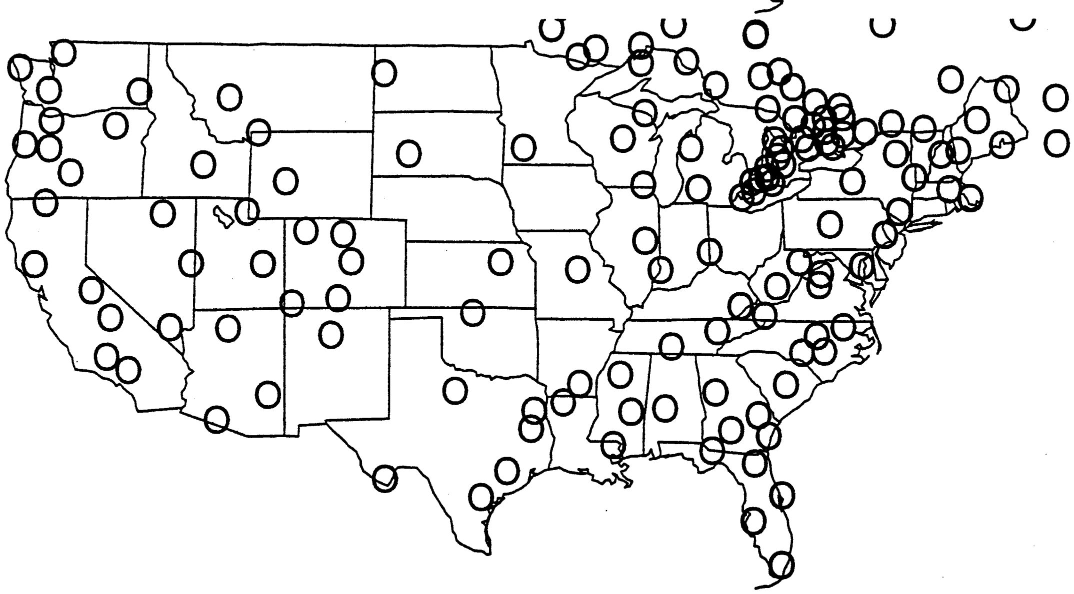
Figure 5: Locations of deleted (X) and not deleted (O) stations using rectangle criterion and smoothing parameter 10.

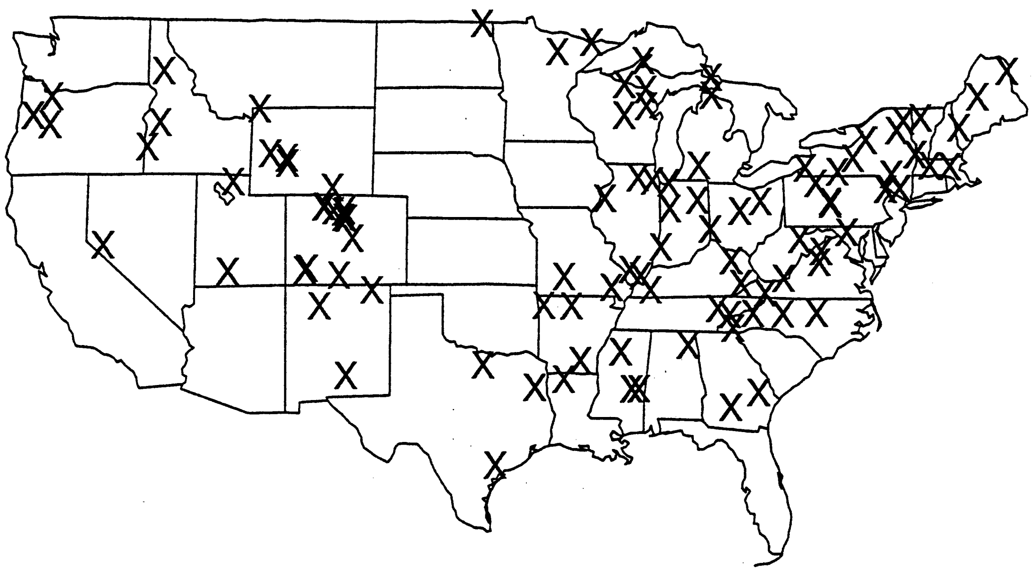
Figure 6: Regional (solid) and rectangle (dashed) criteria scaled to 1 with no stations deleted as a function of the number of stations deleted.

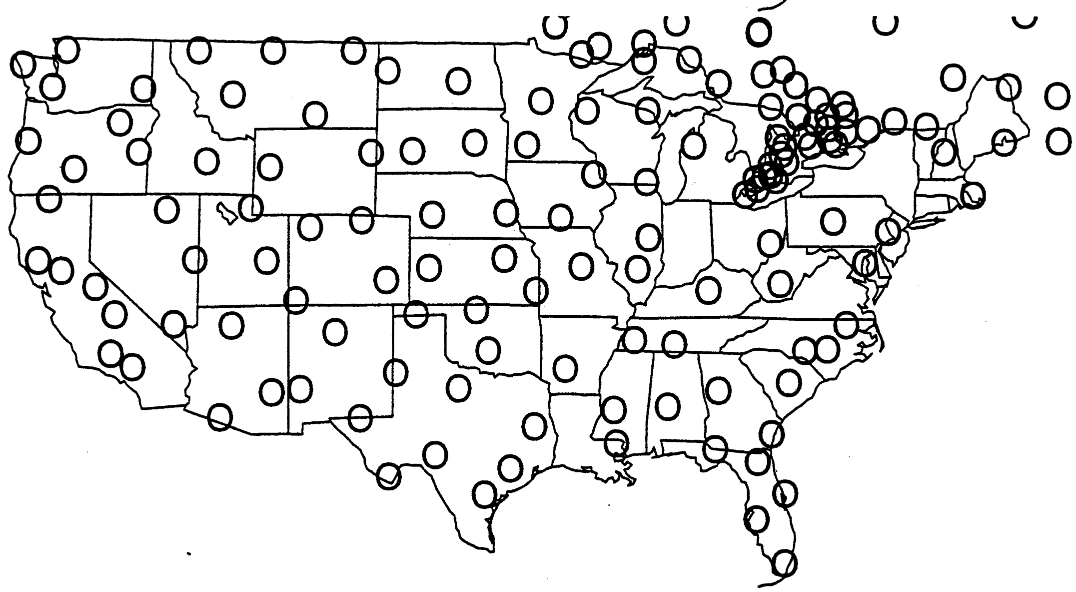


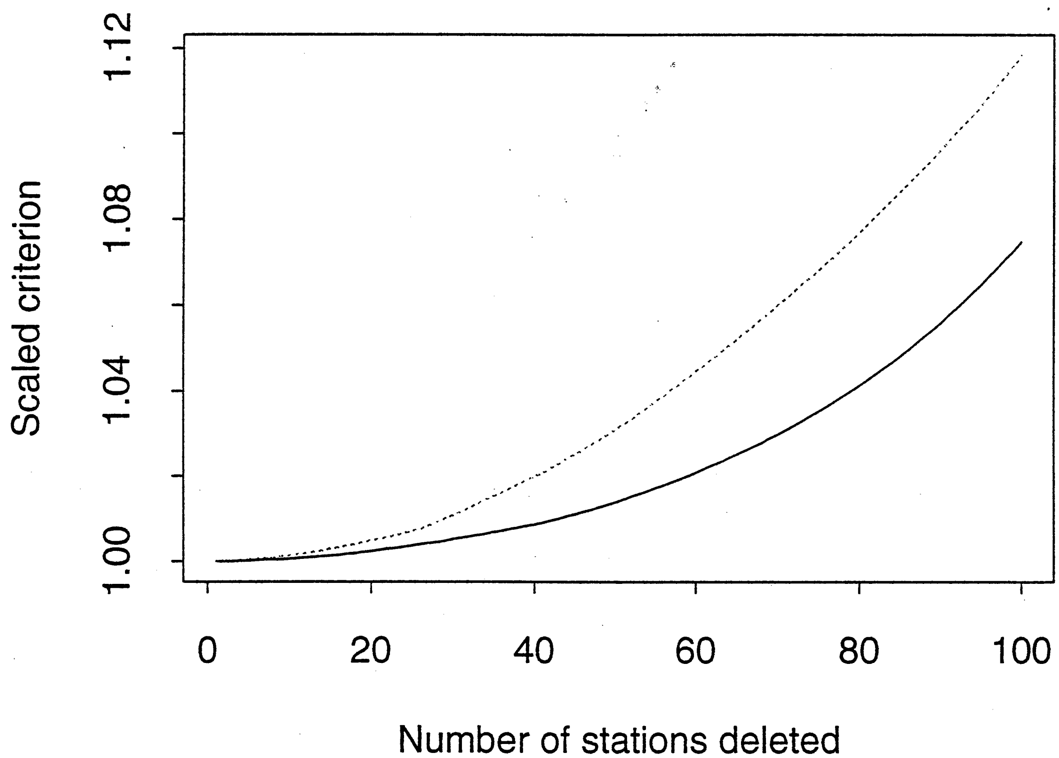



0.0
0.2
0.4
0.6
0.8
1.0
1985
1990
1995
2000
2005
2010
year








Scaled criterion
1.00
1.04
1.08
1.12
0
20
40
60
80
100
Number of stations deleted