Response to Jon Bentley's

"Little Experiments on Algorithms"

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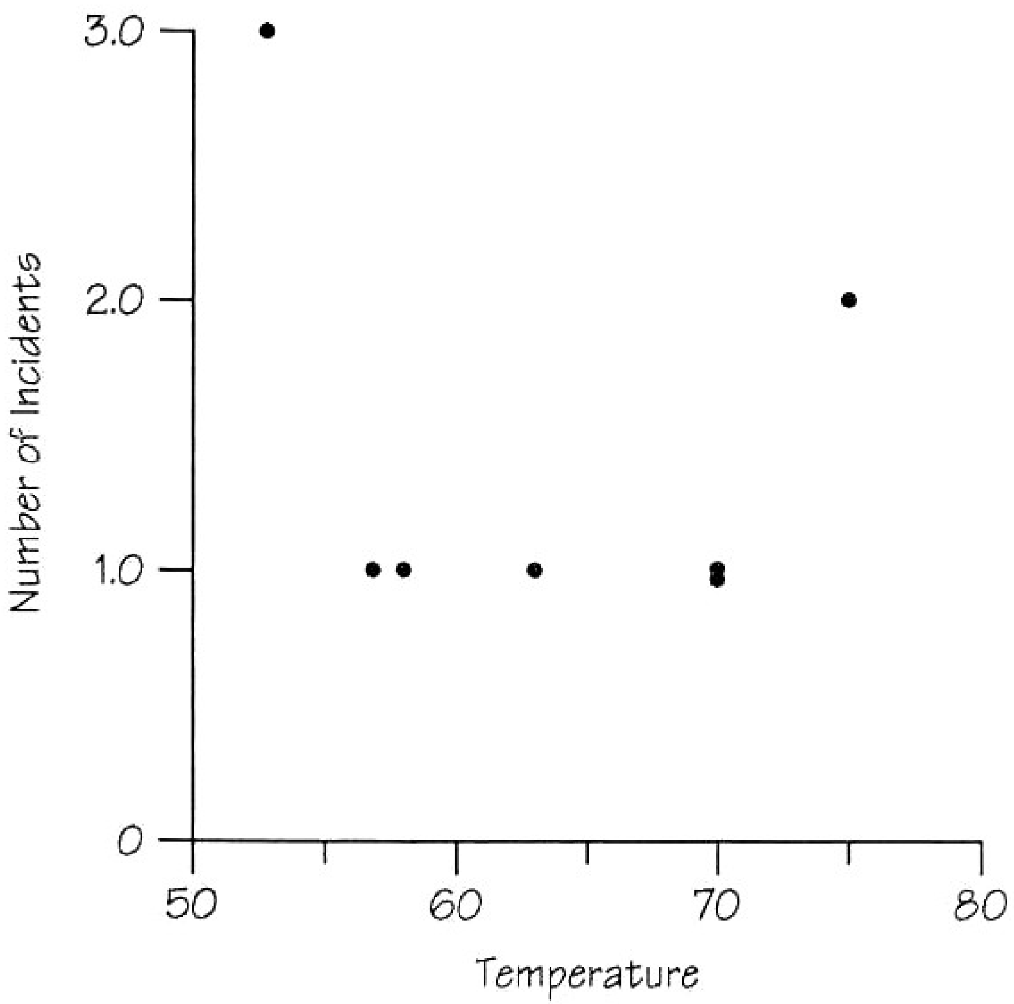
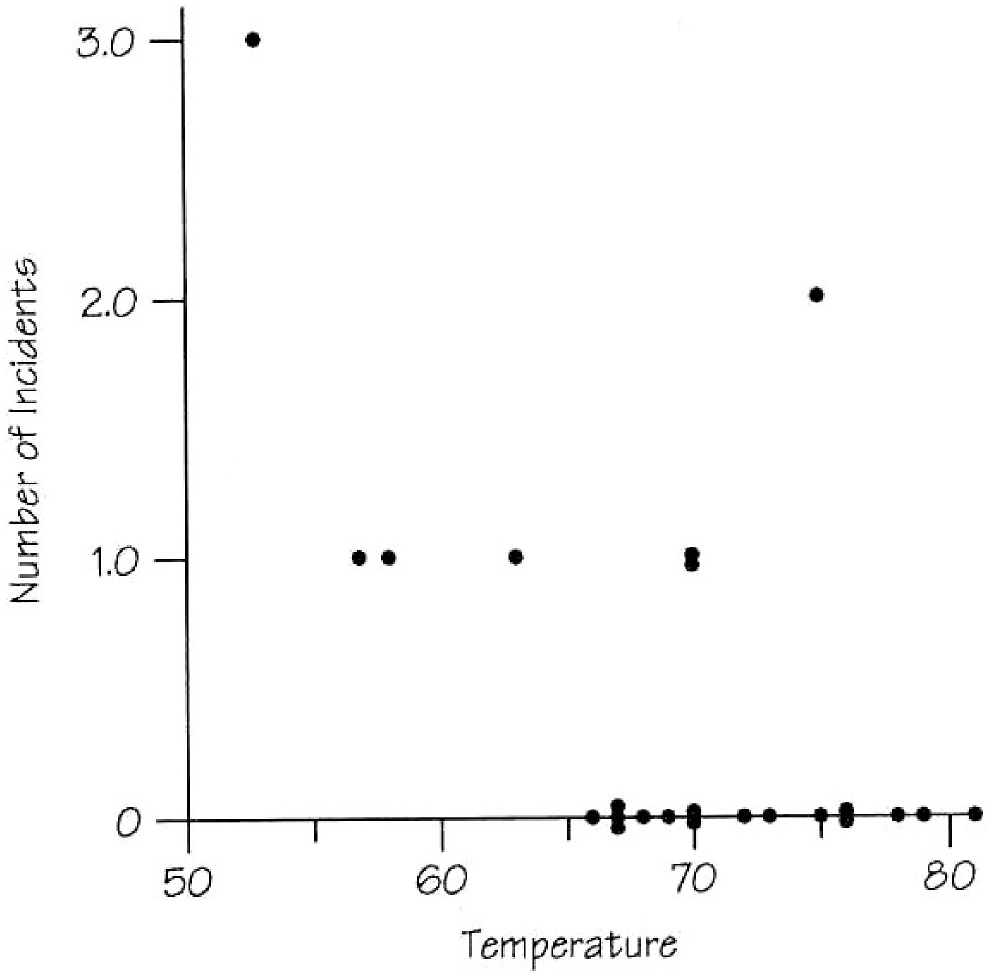
- Jon's examples show value of data analysis in the context of small, careful experiments
- For these processes to be successful, need to:
  - Use substantive knowledge of the problem, along with good ideas about possible models
  - Look at data in different ways, use EDA tools
  - Explore different variables
  - Iterate
- I'm a proponent for this approach to statistics, but there are also other key ideas in statistics that have proven useful
- My Discussion
  - Challenger example
  - Related (?) concepts
  - Network experiments

The Challenger data and scatter plot discussed in a conference call the night before launch.



The mistake was that the flights with zero incidents of damaged O-rings were left off the plot because it was felt that these flights did not contribute any information about the temperature effect. The scatter plot including *all* the data is shown below.



Randomized Trials vis-à-vis Observational Studies and Modeling

- Randomization – great successes in agriculture, medicine, clinical trials
- Currently, economics of development aid … "The basic idea behind the lab is to rely on randomized trials … to study antipoverty programs" (*NY Times*, Feb. 20, 2008)
- Interesting editorial by David Freedman (*Chance*, Winter 2008)
- "Hiccups in data" – variables you didn't know or think about, address via randomization?
- *Question: To what extent do, or do not, the concepts of randomized trials apply to the types of experiments Jon considers?*

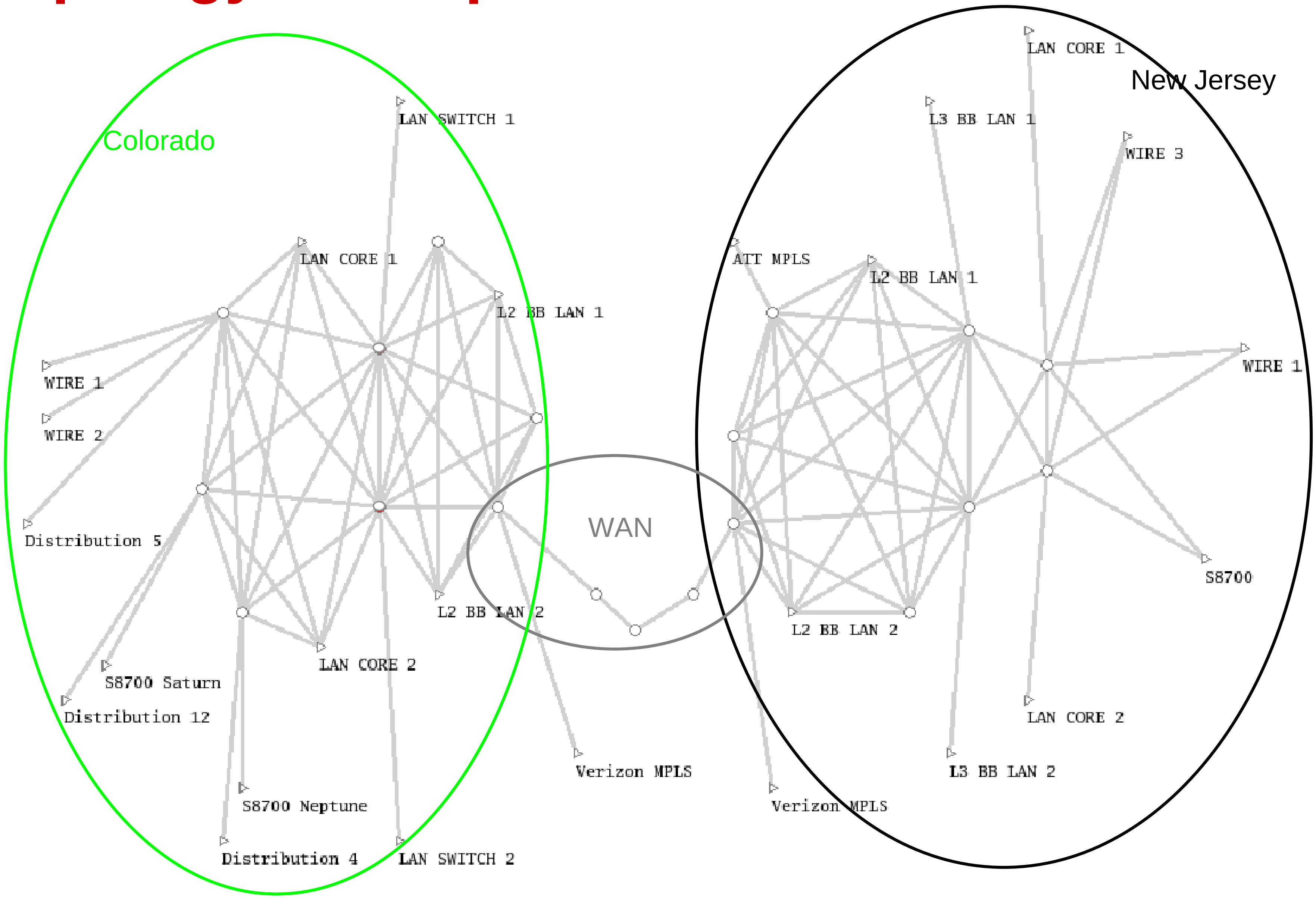
Further Related Statistical Topics

- Blocking
  - Arrange experimental units into groups (blocks) so that the units in a block are similar to one another
  - Try for treatment comparisons within blocks
- Factorial experiments vis-à-vis one-factor-at-a-time experiments
  - Both types have roles
  - Are there situations where there might be interactions, leading to factorial experiments?
  - In order to reduce time to run experiment, do benefits from partial factorials apply?
  - Are purposes related to exploring a response surface, or more to establish/confirm/fit a theory?
- *Question: To what extent do, or do not, these concepts apply?*

Experiments and Networks

- I will mention some issues and relationships to Jon's talk
- Problem Context: VoIP, video over corporate networks
- Goals:
  - Trouble shooting for intermittent problems
  - Provide ability to adapt to changing network conditions in real time, e.g. provide very high QoS and reliability for these applications
  - Evaluating network changes (today)
  - Evaluating possible changes to network equipment and structures (for tomorrow's uncertain applications and traffic)

Topology of Corporate Network



Testing and Experimentation Process

- End-to-end (E2E) tests across network
  - Use phones or special devices for testing
- Path across network for an E2E test does matter, it can change, and it needs to be discovered
- Analyst might or might not be able to select endpoint locations for the E2E tests
  - Sometimes can insert special tests
  - Sometimes use data from phone calls occurring naturally
- Implies that the analysis inherently involves tomography

Mathematical Formulation

- Notation

$$Y_i = \begin{cases} 1 & \text{if test fails} \\ 0 & \text{if test succeeds} \end{cases}$$

$$\mathbf{R} = \text{routing matrix}$$

$$R_{ij} = \begin{cases} 1 & \text{if the } j^{\text{th}} \text{ node (or edge) is involved in the } i^{\text{th}} \text{ test} \\ 0 & \text{otherwise} \end{cases}$$

$$\text{for } j = 1, \ldots, K \text{ and } K \text{ is total number of nodes (or edges)}$$

$$\pi_i = \text{probability the } i^{\text{th}} \text{ test fails}$$

$$p_j = \text{probability that performance on this node fails}$$

$$p = \text{probability test fails although none of the nodes directly "causes" failure}$$

Then

$$Y_i \sim \text{Bernoulli}(\pi_i)$$

Probability of Success

- End-to-end test success implies success on each traversed node and also for the background

$$(1-\pi_i) = (1-p) \times \prod_{\{j:R_{ij}=1\}} (1-p_j)$$

- Log likelihood

$$C + \sum_{\{i:y_i=0\}} [log(1-p) + \sum_{\{j:R_{ij}=1\}} log(1-p_j)] + \sum_{\{i:y_i=1\}} log(1-[(1-p) \times \prod_{\{j:R_{ij}=1\}} (1-p_j)])$$

Problems

- Likelihood a complicated non-linear expression
  - Could be intractable
  - Heuristics useful
- More fundamentally, truth can be on boundary of parameter space so standard asymptotic theory does not apply
- Hence, is likelihood analysis a good approach?
- *Real networks are large and raise complicated issues such as these in spades, so perhaps we should study the topic using Jon's ideas with careful, appropriately selected "small network experiments"?*

Summary

- Congratulations to Jon for
  – His series of careful “small experiments”
  – For using data analysis creatively and successfully to gain insights
  – And for promoting the approach
- I’ve tried to raise questions on how, and whether or not, other statistical topics fit into experimentation in this domain
  – Randomized trials
  – Blocking variables
  – One-variable-at-a-time vis-à-vis factorial experimentation
  – Observational data and modeling vis-à-vis randomized controlled experiments
- Networks offer a fruitful area for pursuing these topics

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