Statistical Models and the Financial Crisis: (our share of the blame)

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What Volatility?*

*“What is considered to most observers is not so much the annual returns-which though considered somewhat high for the strategy could be attributed to the firm’s market making and trade execution capabilities-but the ability to provide such smooth returns with so little volatility” ***Madoff Tops Charts; sceptics ask how*, by M. Ocrant, MAR/Hedge 89, May 2001.**

Credit Derivatives

- **Credit risk:** *distribution* of loss due to failure of a financial agreement.
- **Credit derivative:** security which allows the transfer of credit risk from one party to another
- **Credit Default Swap (CDS):** in the event of *default*, the protection seller compensates protection buyer for loss. In return, buyer makes (quarterly) payments of the *swap spread*. (62 trillion in notional, Jan 2008)

CDS

CDO

Collateralized debt obligation (CDO) A portfolio of fixed-income assets or CDSs (synthetic) typically divided into different tranches: senior tranches (AAA), mezzanine tranches (AA to BB), and equity tranches (unrated).

The **notional** amounts of over-the-counter derivatives continued to expand in the first half of **2008**, according to data from the Bank for International Settlements (BIS). They reached $684 trillion at the end of June, 15% higher than in December 2007. Interest-rate contracts, which account for the lion's share of the market, expanded by 17%. *Economist Nov 29, 2008*

Risk and Reward

Standard Risk Measures

- **Standard Deviation** or Variance $E(\text{Loss}-\mu)^2$
  or one-sided (semi) variance $E[(\text{Loss}-\mu)^2 | \text{Loss} > \mu]$
- **Value at Risk:** Confidence level *α=95% or 99%:*
  VaR $= \min\{x;\ P[\text{Loss} \le x] \ge \alpha\}$. (Quantile)
- **Conditional Tail Expectation** CTE $= E[\text{Loss}|\text{Loss}>\text{VaR}]$
  (or Tail-VaR, Tail Conditional Expectation TCE, or Expected Shortfall)
- **Risk-adjusted Return**

$$Sharpe = \frac{E(R) - benchmark}{SD(R)}$$

Risk in a VaR-Regulated environment

- Suppose we apply **VaR constraint** e.g. $VaR_{0.99}=c$ and allow investors to trade derivatives so as to modify the loss distribution in the tail, (subject to constant mean, with returns i.i.d.). Then provided trader's incentives monotone in E(Return|Return>barrier) , then CTE→∞ .
- **VaR can be gamed:** trade very large risks with low probability for increased regular returns

Risk Mismanagement(2009) JOE NOCERA *The New York Times Magazine.* "Guldimann, the great VaR proselytizer, sounded almost mournful when he talked about what he saw as another of VaR's shortcomings. To him, the big problem was that it turned out that VaR could be gamed. That is what happened when banks began reporting their VaRs. To motivate managers, the banks began to compensate them not just for making big profits but also for making profits with low risks. That sounds good in principle, but managers began to manipulate the VaR by loading up on what Guldimann calls "asymmetric risk positions."*

VaR Increases Risk: Boyle, Hardy& Vorst ; **Sharpening Sharpe Ratios**, Goetzmann, Ingersoll, Spiegel, Welch; Also Bernard and Tian; Basak and Shapiro

*

Example: A default process

- The default process of firm i is a (random) stopping time τ_i with respect to some filtration* $\mathcal{F}_t$ so that the event $[\tau_i > t] \in \mathcal{F}_t$ for all t.
- Easy to generate random default events with given cumulative distribution function F using $\tau_i = F^{-1}(U)$ or

U[0,1] r.v.

$\tau_i = F^{-1}(\Phi(Z_i))$ where Z_i is N(0,1) and Φ is the standard normal c.d.f.

- When the default occurs, we need to model loss given default. (easier?)

* The filtration may depend on what information investors are assumed to have at time t. Guo, Jarrow & Zeng ; Duffie & Lando ; assume investors have incomplete and lagged information at discrete time points and show structural models can be viewed also as reduced-form.

Incorporating Dependence("industry Standard" Gaussian copula model*)

- Generate $\tau_i = F_i^{-1}(\Phi(Z_i))$ where Z_i are (dependent) N(0,1) and Φ is the standard normal c.d.f. . Z_i share factors, e.g.

$$Z_i = \rho_i M + \sqrt{1-\rho_i^2}\,\varepsilon_i$$

where ε_i are independent N(0,1) *idiosyncratic risk* factors and

M is a common (unobserved) N(0,1) market risk factor.

Issues:

- **Implementation:** easy; effect dimensionality=# common factors
  - Easy to introduce covariates, additional (latent) factors, etc.
- **Fit** to market data; Gaussian copula(ρ>100%). Does not allow "contagion" (past defaults directly influence probability of future defaults) or continuous time model. **One time horizon.**
- Parsimonious but **rigid correlation structure: Gaussian copula induces too much independence especially in tails** (Embrechts et al.)
- Distribution of loss given default (or recovery rate) independent?

* Li, D. (2000) On default correlation: a copula function approach. *J. Fixed Income*, 43–54

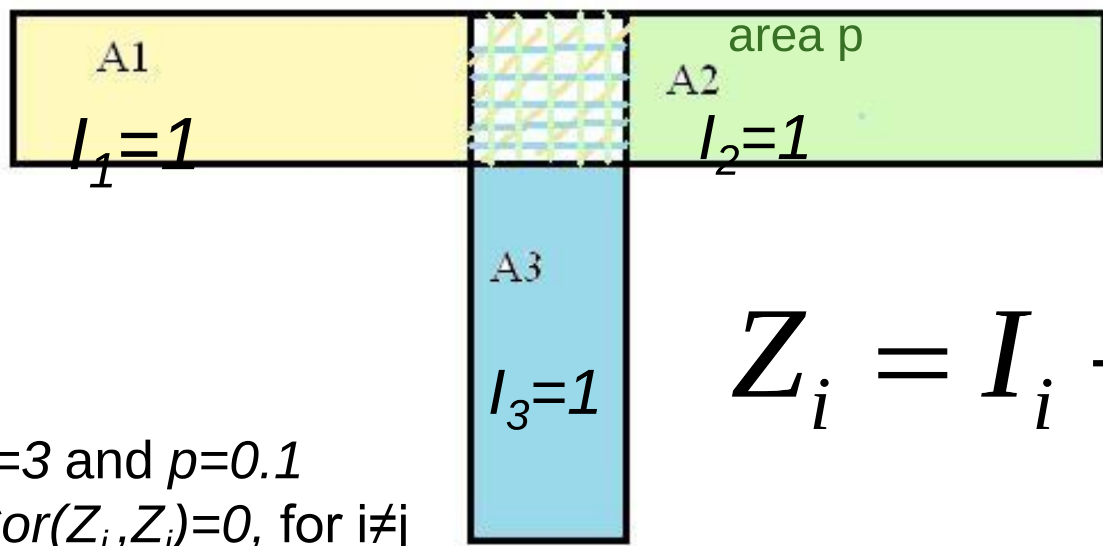
Default dependence "captured" with default Correlation

Highly Ambiguous Concept

- The correlation between the default times of two firms:
- The correlation parameter in a Gaussian copula
- The correlation between two firm default indicators over a fixed time horizon (Depends on time horizon, maximum possible value depends on probabilities, may be <<1)
- Correlation under extreme conditions changes anyway (panic and margin calls)
- **Correlation does not determine the joint distribution. Not even close!**
- The correlation between the distances to default (V_t-D_t assuming BM) (to be described)

Pairwise Correlations Misleading Measure of dependence

All have area p



$$Z_i = I_i + \varepsilon_i$$

c=3 and *p=0.1*

$Cor(Z_i, Z_j)=0$, for i≠j

Probability one firm defaults=.0035

Probability all three firms default is *=0.00125*

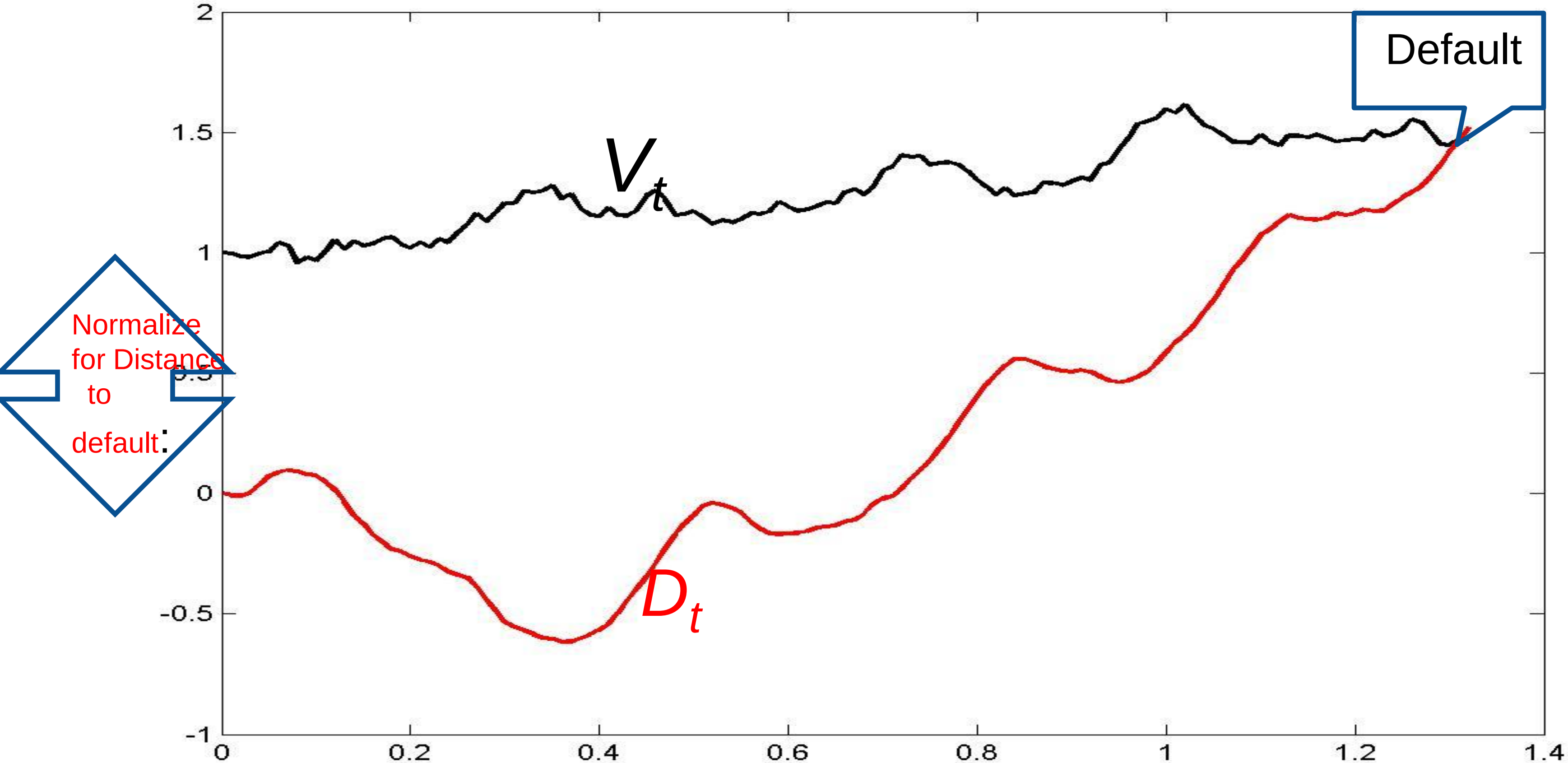
twenty nine thousand times as large as in independent case

Dynamic Models: Structural

- The gross market value firm's assets process V_t (which may or may not be observed by investors) is modelled, e.g. using a a (jump) diffusion. (Merton's model: on log scale, V_t BM, variance σ^2)
- Default threshold D_t (liabilities) modelled as deterministic function or diffusion or integrated diffusion. The firm defaults at time $\tau=\inf\{t;\ V_t \leq D_t\}$
- V_t (and/or D_t) may be modelled with covariates, latent variables, missing information or discrete observations. Often D_t =non-random.

Merton; Black and Cox; Zhou; default of a company at first time when the firm-value falls below default boundary. Multivariate extensions -Hull et al. ; Overbeck/Schmidt

Structural Model



Multivariate Case:

- $V_t^{(i)}$ modeled as diffusion (or (Geometric) Brownian motion)

$$dV_t^{(i)} = \mu^{(i)}\, dt + \sigma^{(i)}\, dW_t^{(i)}$$

constants

- $D_t^{(i)}$ stochastic or deterministic function of t
- $W_t^{(i)}$ are correlated Brownian Motion
- Default of name (i) is first passage time of firm $V_t^{(i)}$ *to default barrier* $D_t^{(i)}$

Issues for Structural Models

- **Advantages:** structural models permit using obervable covariates; Book values, equity prices, leverage, debt, ….
- **Implementation:** only in special cases (1-2 dimensional case) are there closed form joint distributions for hitting times.
- **Calibration:** needs Monte Carlo-difficult in 125+ dim
- **Fit to market data**. Defaults predictable (default rate near t=0 is zero*) (Resolved by adding Jumps and/or delayed filtration.
  - For "*contagion**", (past defaults directly influences probability of future defaults), increase intensity of jumps after defaults.)
  - Regime specific dependence on stock returns/volatility**

* Duffie and Lando; Guo, Jarrow and Zeng

** Alexander & Kaeck

An Alternative

$$dV_t^{(i)} = \mu^{(i)}(M_t)\,dt + g^{(i)}(\sigma_t)\,dW_t^{(i)}$$

Independent BM *

Market factors M_t, σ_t satisfy mean-reverting (CIR-like) diffusion relationship e.g. $\mu^{(i)}(M_t) = \beta^{(i)} M_t, \quad g^{(i)}(\sigma_t) = \xi^{(i)} \sigma_t$

$$dM_t = \kappa_M(\theta_M - M_t)\,dt + h(M_t)\,dW_t^{(M)}, \quad h(x) = 1 + a|x-\alpha| + b(x-\alpha)$$

independent

Can add Jumps to permit non-zero yield spreads (failure rate) at t=0 and model default contagion

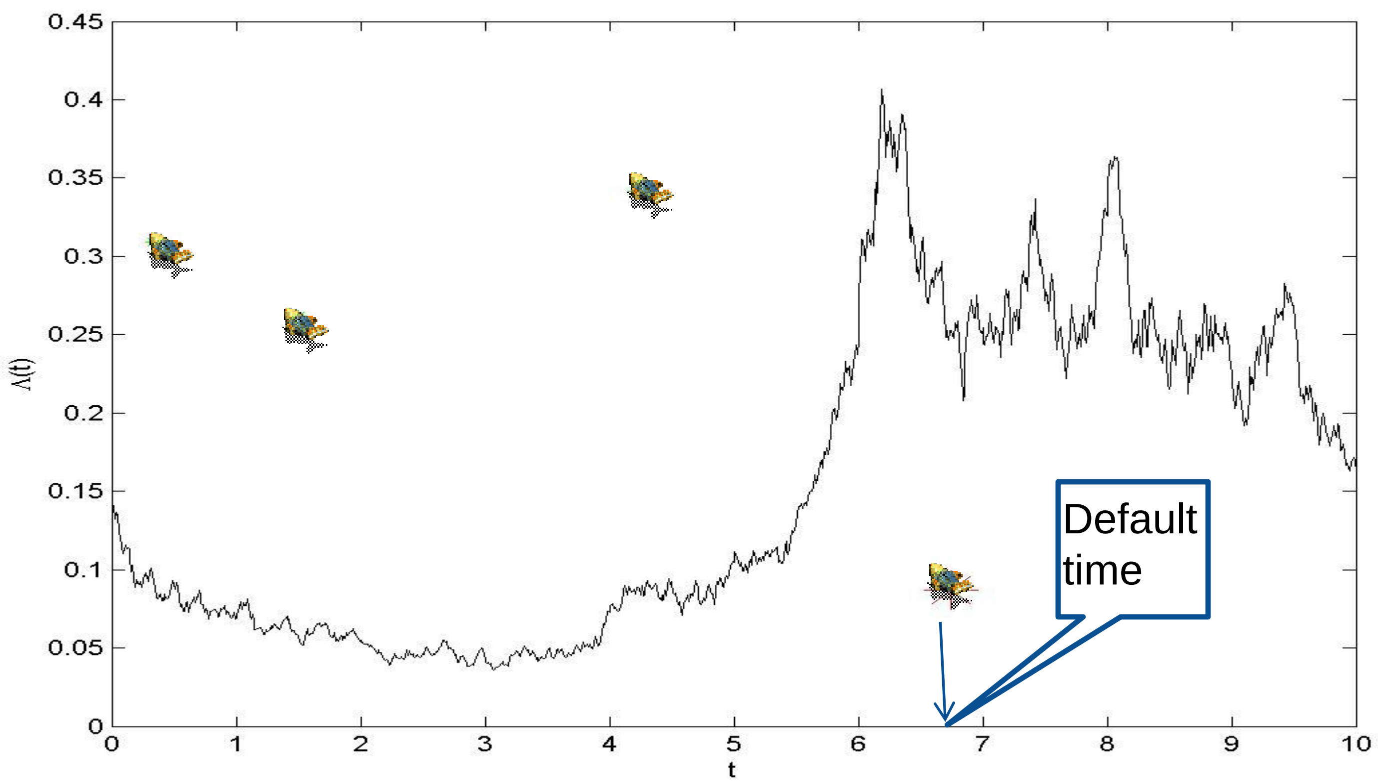
Equivalent to first passage time of a time-changed Brownian motion, with time change $T(t) = \int_0^t \sigma_t^2 dt$ to barrier at integrated market process $D_t = d_0 + \int_0^t M_s ds$

- Advantage that we can simulate M_t, σ_t then calculate barrier crossing probabilities conditionally
- Calibrates well across different tranches/maturities

•A Multiname First-Passage Model for Credit Risk, A. Metzler and D. McLeish
http://www.watrisq.uwaterloo.ca/Research/2009Reports/09WatRISQReports.shtml

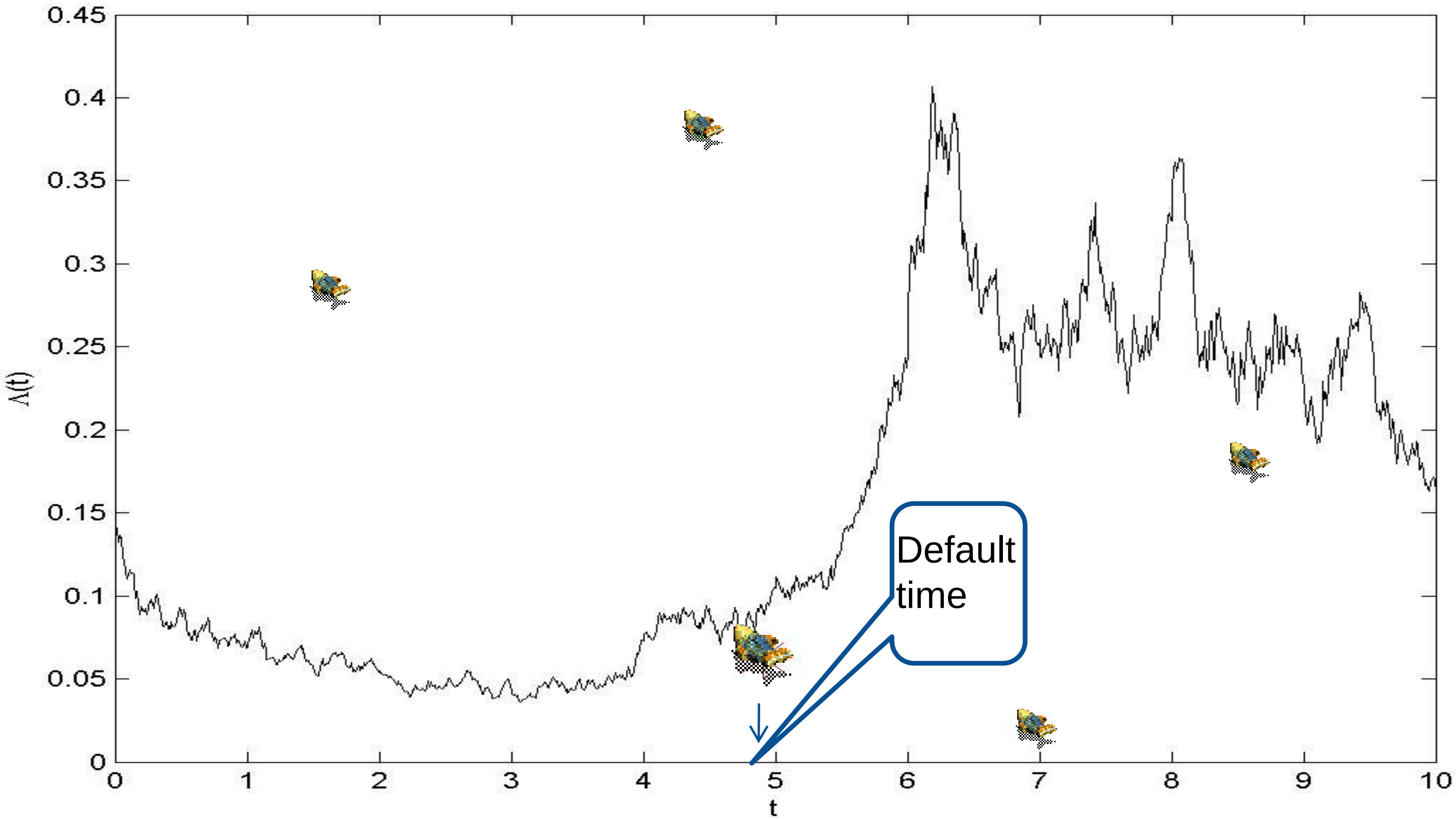
Reduced-form (hazard rate) Models*

Defaults are generated according to a non-homogeneous Poisson Process whose intensity process (the risk factor process) satisfies a jump diffusion:
P[default in $(t,t+\Delta t)$]= $\Lambda(X_t)\,\Delta t$

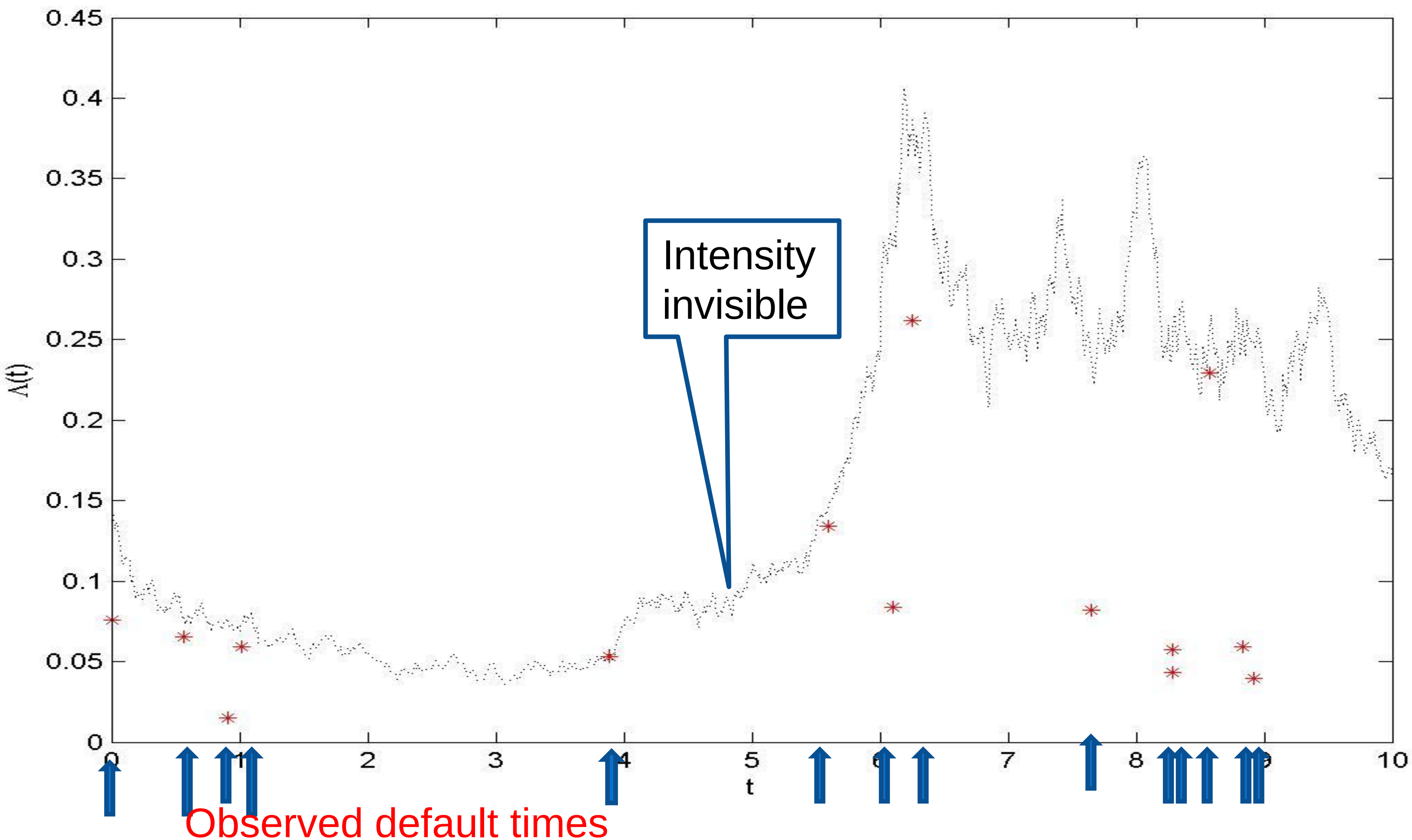


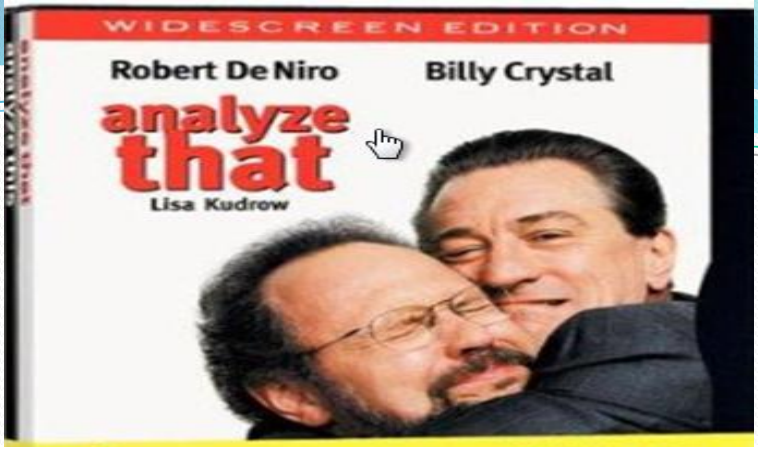
*Jarrow and Turnbull (1992,1995) , Longstaff and Schwartz (1995), Duffie and Singleton (1999), Hull and White (2000) , Gieseke (2008), Litterman and Iben (1991), Madan and Unal , Lando , Duffie and Singleton, and Duffie., Jarrow, Lando, and Turnbull (1995)

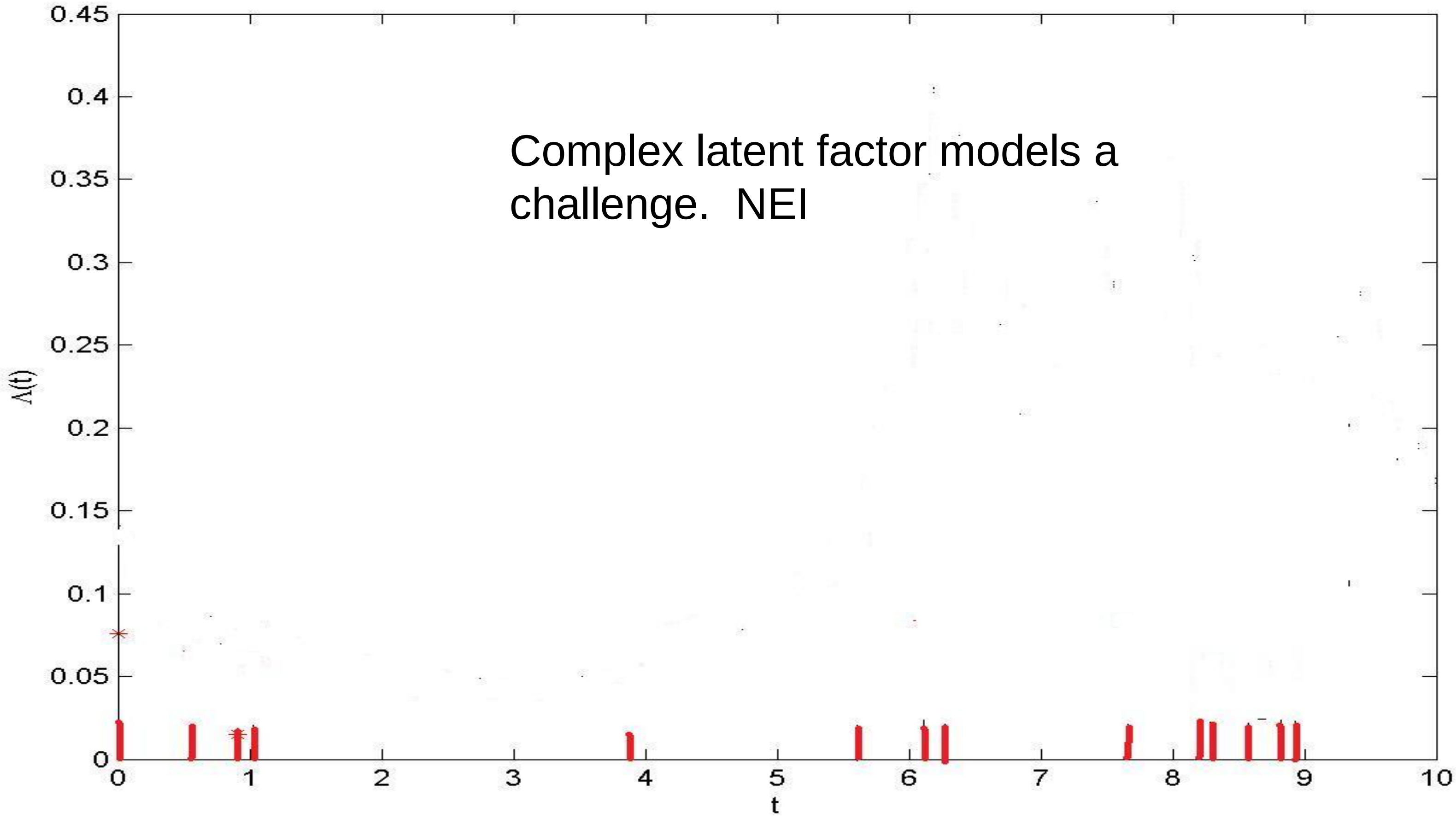
Reduced form, Firm 2



Observations: a number of defaults

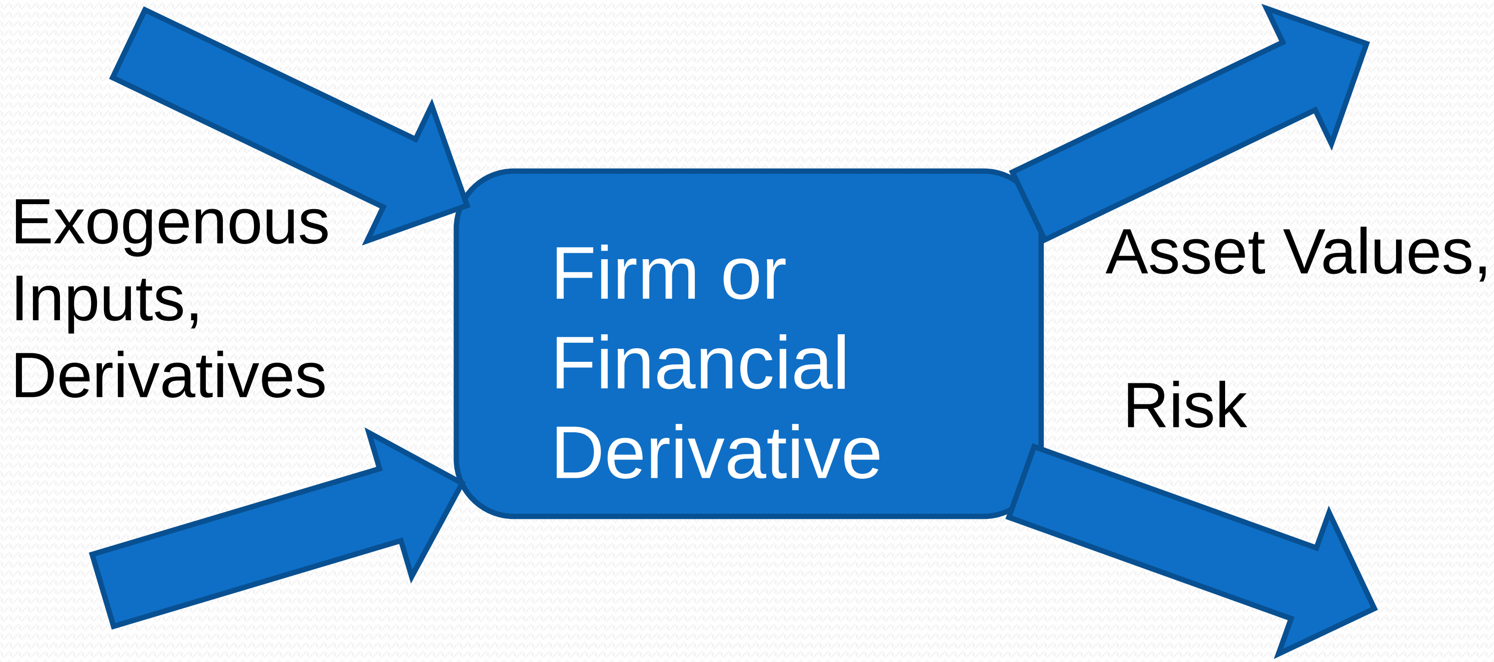




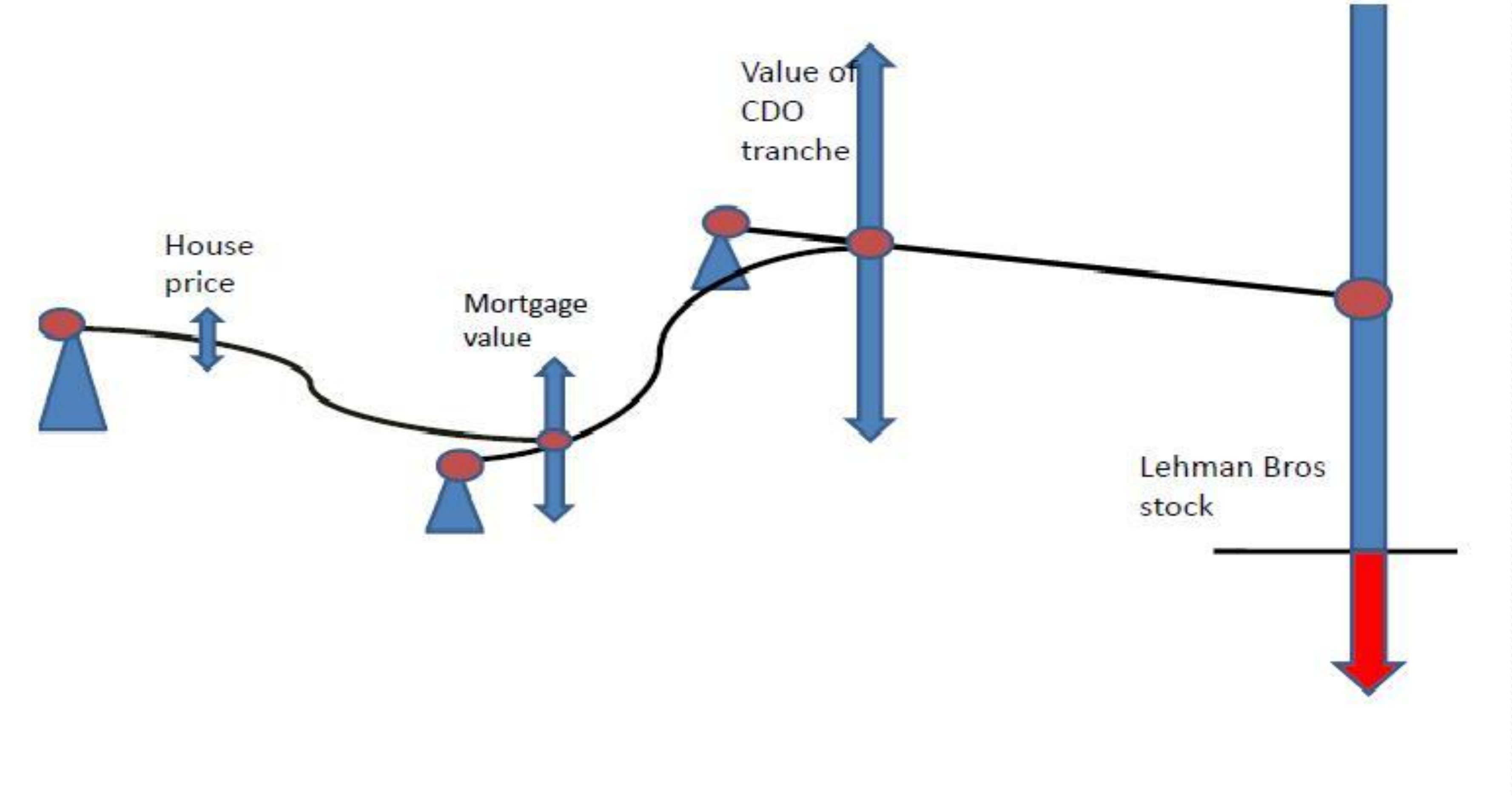


Complex latent factor models a challenge. NEI

The way it is (once was?)



or more like....


House
price
Mortgage
value
Value of
CDO
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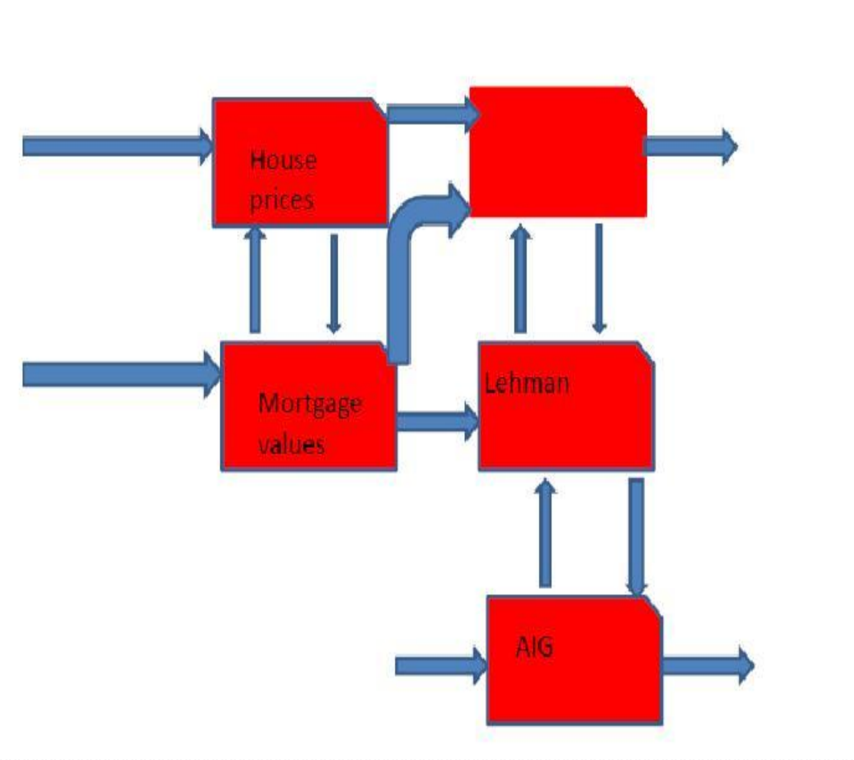
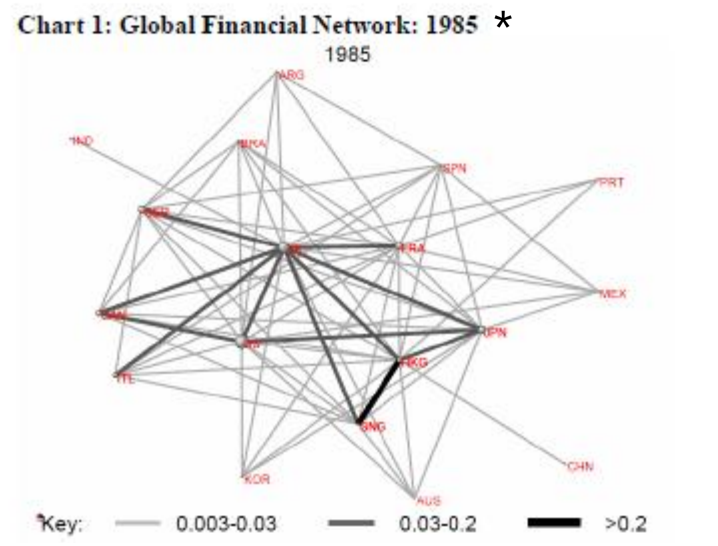
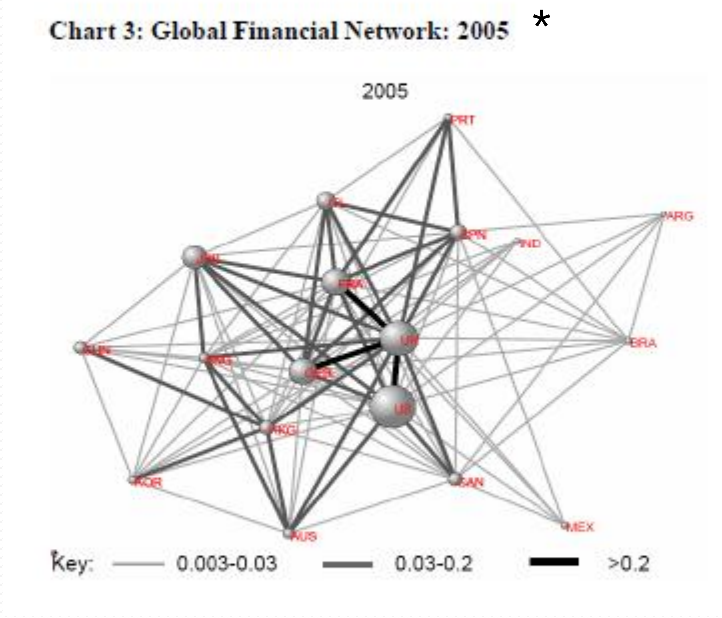
Lehman Bros
stock

Or………

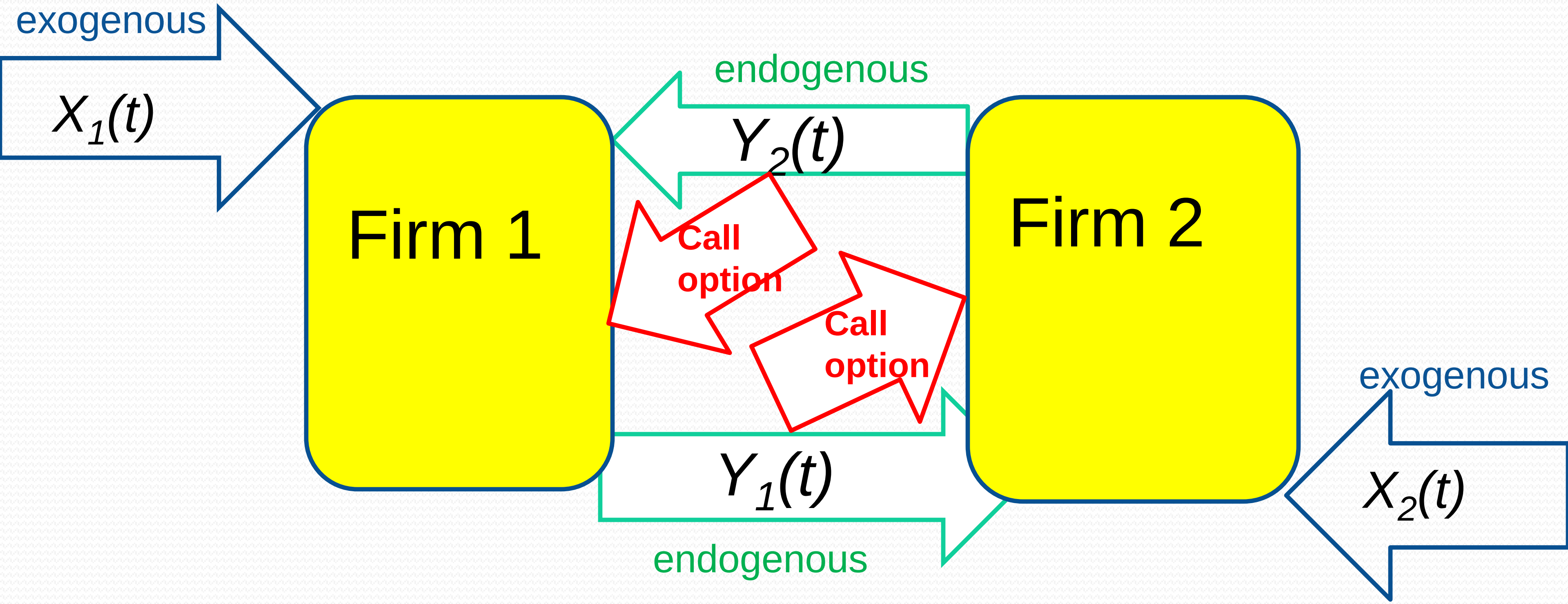






**** From RETHINKING THE FINANCIAL NETWORK,*** Andrew G Haldane
http://www.bankofengland.co.uk/publications/speeches/2009/speech386.pdf

A Simple Network Simulation



Each Firm maximizes risk adjusted return

Four possible investments for Firm 1(Firm 2 similar)

- Exogenous asset (stock) $X_1(t)$ (GBM)
- Endogenous stock on Firm 2 , $Y_2(t)$
- At-the-money call option on $Y_2(t)$
- Risk free asset (0 return)

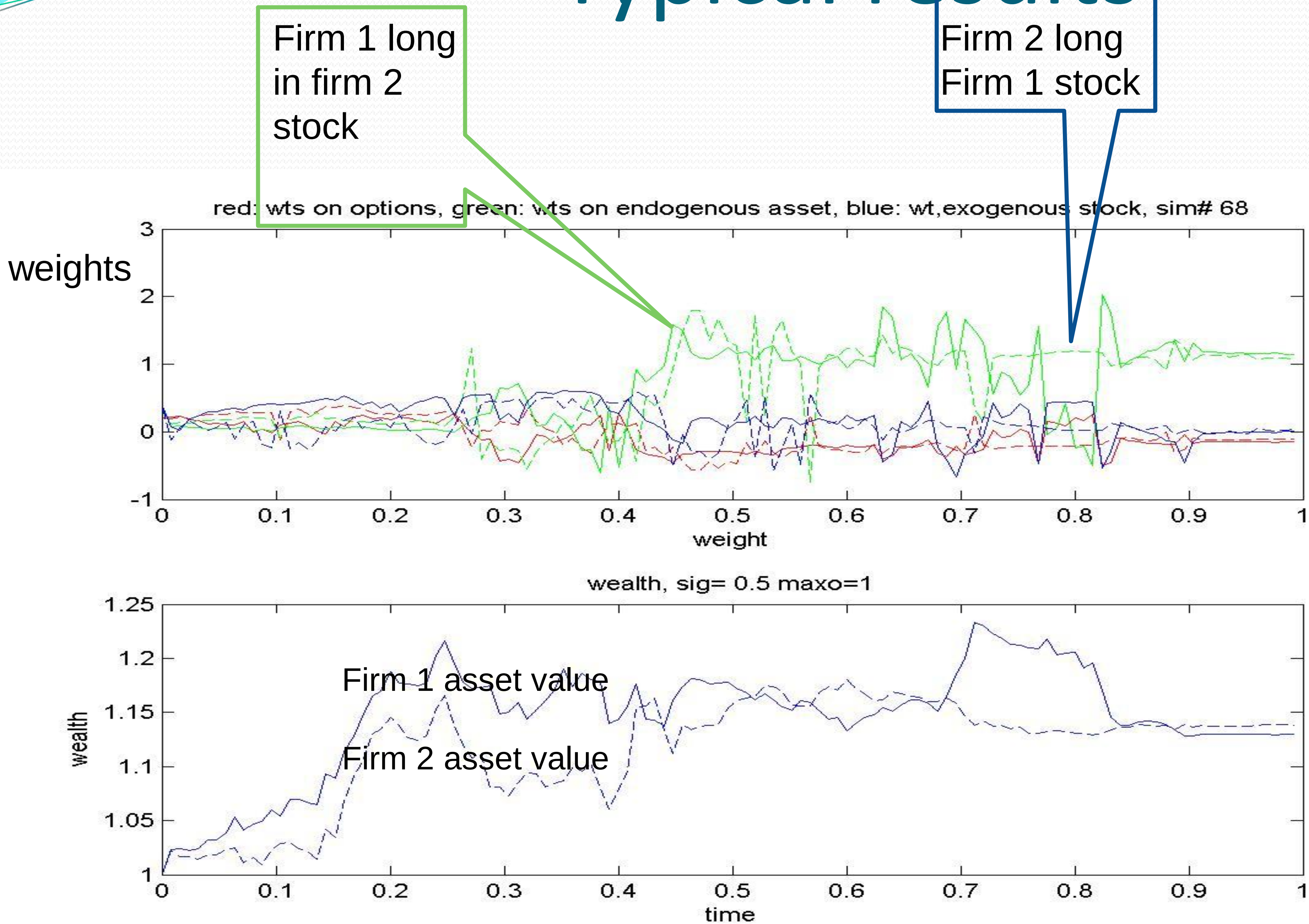
Choose weights on these assets to maximize risk-adjusted returns

$$Sharpe = \frac{E(R)}{\sqrt{Var(R)}}$$

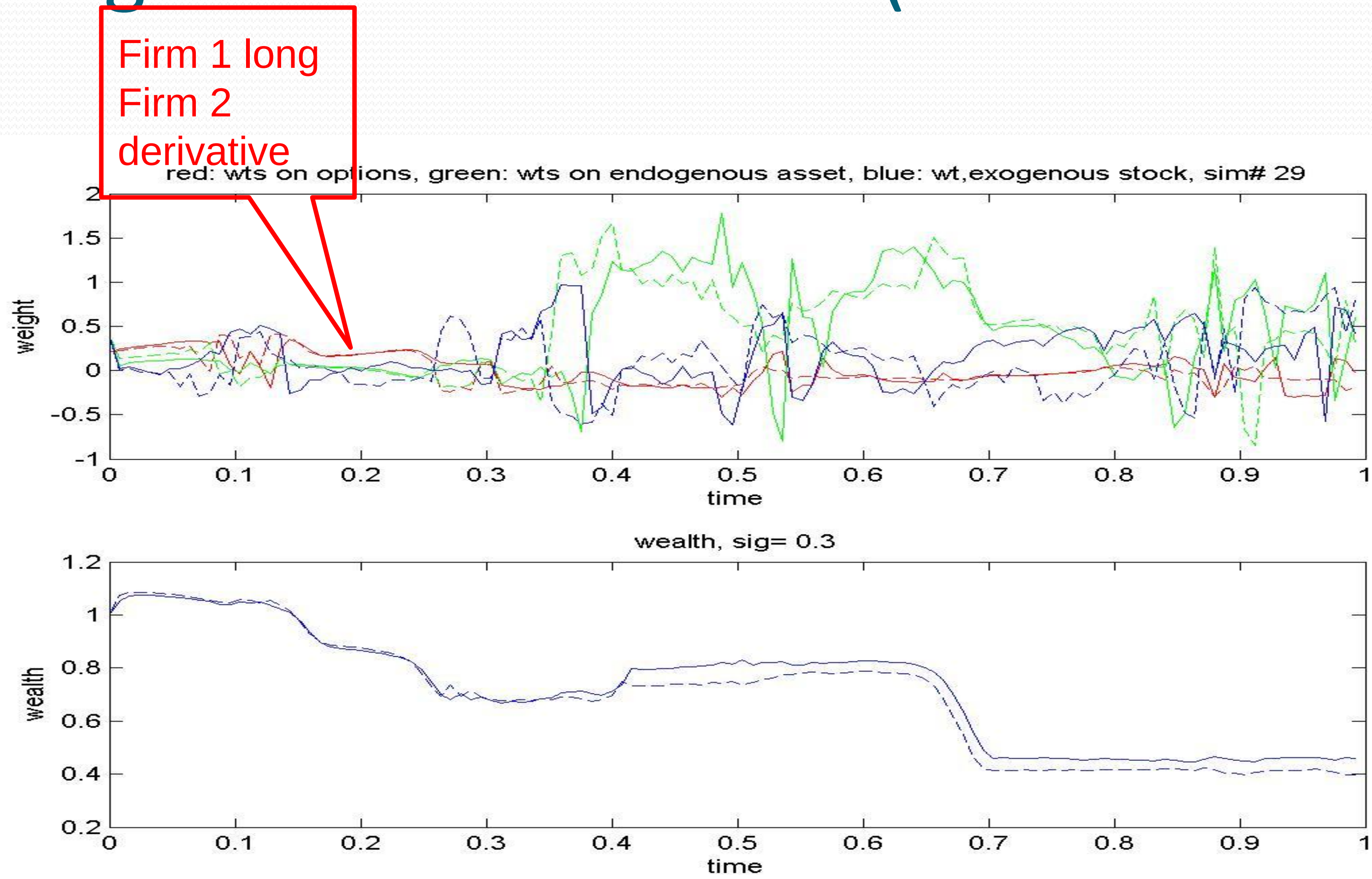
Subject to constraint: daily VaR=4%.

Means and Variances exponentially reweighted average of past values obtained from 50 observations/day

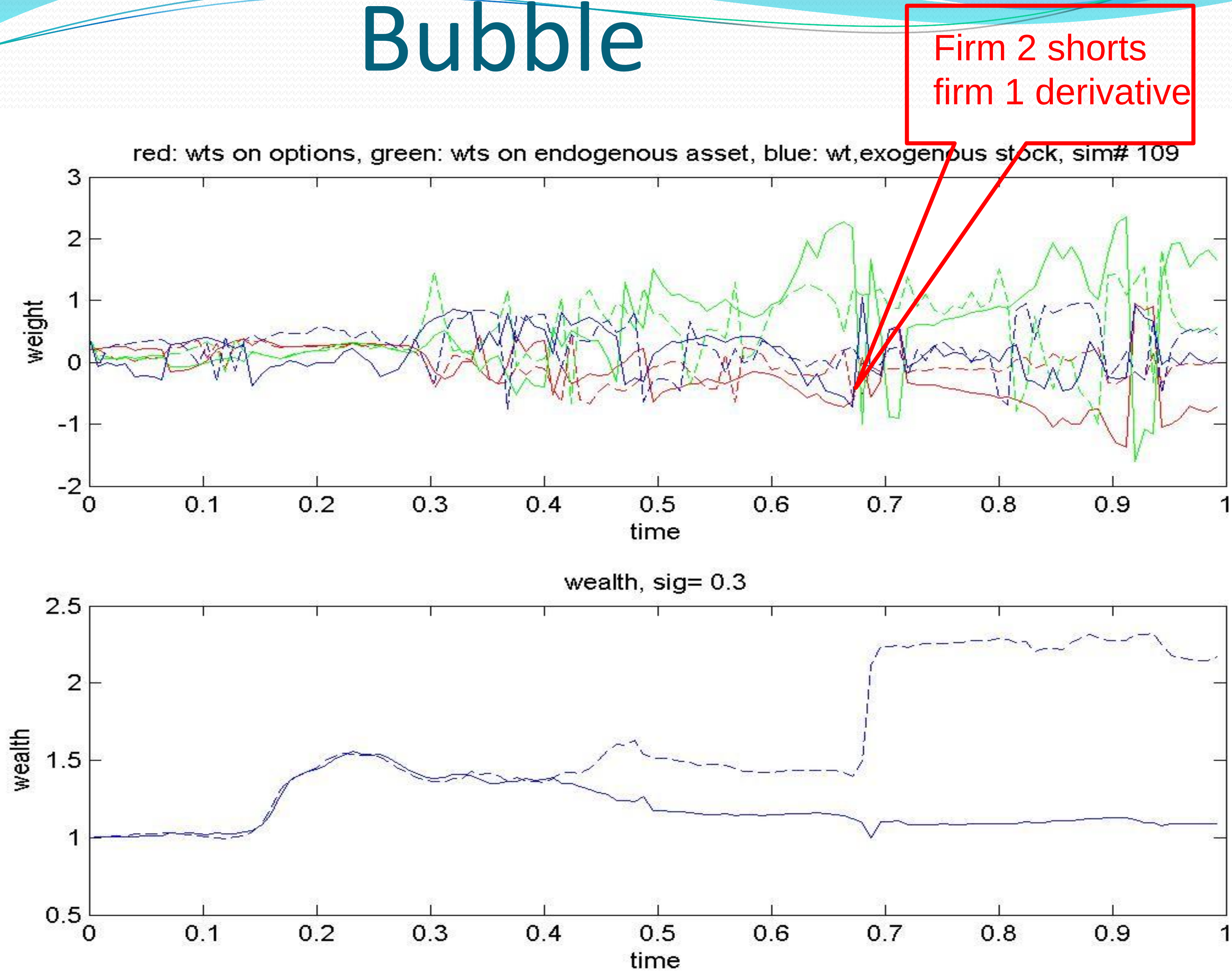
Typical results



Weights and Firm Values (bust scenario)



Bubble


Firm 2 shorts
firm 1 derivative
red: wts on options, green: wts on endogenous asset, blue: wt,exogenous stock, sim# 109
weight
time
wealth, sig= 0.3
wealth
time

Probability of bubble/bust

- **Parameter values:** drift of exogenous stock=5% , MER of firms=5%, r=0, horizon=1 year, bust=50% decrease, bubble=100% increase, volatility: $.01 < \sigma < .5$
- Feedback increases probabilities of bust/bubble by factor of approximately 10 over independent case.
- Similar if |weights|<1.
- Network has a life (and death) of its own.

Conclusions

- Too much attention paid to constructing models with **analytic tractability.** (Monte Carlo)
- **Dynamically changing models (clock speed and Google effect)** speed of information flow- faster by orders of magnitude. Parameters /model change to reflect changes in volatility, correlation, feedback under extreme conditions, leverage effects, technology **.** Does the firm-value process of a firm near default differ from standard model? (Does an octogenarian invalid differ from a 21 year-old)? Predation, liquidity, fear factor.
- **Network model feedback :** (Finance is not Physics: endogeneity) psychology of defaults. Institutional aspects, leverage, margin calls, local optimization.
- **Traders as Gods.** Calibration vs. Estimation. Error estimates and sensitivities. Complexity of multivariate distributions. (Even in normal case, 1% change in ρ = 50% change in vol when N=125)
- Extremes of portfolios are driven by **tail dependence***. Modelling (tail) dependence* (has psychological component)

*Revisiting the edge, ten years on (2008) Chavez-Demoulin, Embrechts

- **Linear Models:** Extraordinarily useful under "normal" conditions, they place too much faith in their own ability to forecast under extreme conditions. ***Where is the independence*?** (idiosyncratic factors). Statistical models require identifying i.i.d. residuals (hard in finance)
- **For risk management, unique model unnecessary.**

We use a battery of tests for random number generators. As important to subject risk strategies to a **portfolio of tests** motivated by competing models, driven by randomly generated scenario paths/parameters.

Edmund Phelps, (won the 2006 Nobel prize for economics) "Risk-assessment and risk-management models were never well founded. There was a mystique to the idea that market participants knew the price to put on this or that risk. But it is impossible to imagine that such a complex system could be understood in such detail and with such amazing correctness…the requirements for information…have gone beyond our abilities to gather it."*

* **Plato's cave** *The Economist*, Jan. 22 2009.

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