Robust inference in two-phase sampling with application to unit nonresponse

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Outline of the presentation

1. Introduction
2. Measuring the influence: the conditional bias
3. Robust estimators
4. Application to unit nonresponse
5. Simulation study
6. Concluding remarks

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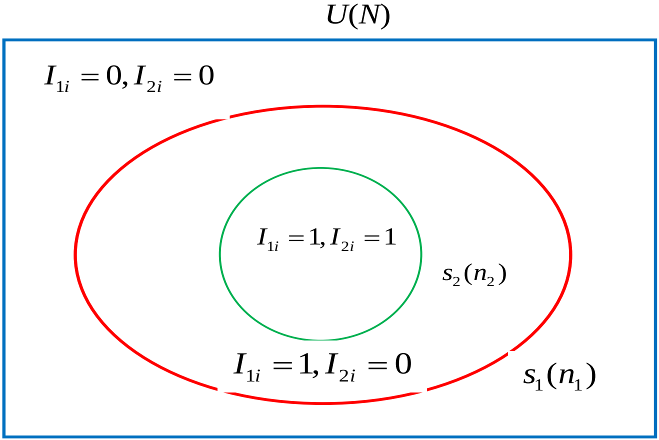

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- $\hat{Y}_{DE}$ is design-unbiased for Y; that is,

$$E_1 E_2(\hat{Y}_{DE} | \mathbf{I}_1) = Y$$

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- The total error of $\hat{Y}_{DE}$:

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- How to construct a robust estimator to the presence of influential units? Single phase designs: Beaumont, Haziza and Ruiz-Gazen (2011).

Measuring the influence: the conditional bias

- We distinguish between three cases:
  - $i \in s_2$: sampled unit
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Measuring the influence: the conditional bias

- Arbitrary two-phase design:

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B_i^{DE}(I_{1i}=1, I_{2i}=1) &= \underbrace{\sum_{j\in U}\left(\frac{\pi_{1ij}}{\pi_{1i}\pi_{1j}}-1\right)y_j}_{\text{Influence of unit } i \text{ on the first-phase error}} \\
&+ \underbrace{\sum_{j\in U}\frac{\pi_{1ij}}{\pi_{1i}\pi_{1j}}\left(\frac{\pi_{2ij}}{\pi_{2i}\pi_{2j}}-1\right)y_j}_{\text{Influence of unit } i \text{ on the second-phase error}} \\
&= \underbrace{\sum_{j\in U}\left(\frac{\pi^*_{ij}}{\pi^*_i\pi^*_j}-1\right)y_j}_{\text{Total influence of unit } i}
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- SRSWOR/SRSWOR: $\pi_i^* = \frac{n_1}{N} \times \frac{n_2}{n_1} = \frac{n_2}{N}$

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B_i^{DE}(I_{1i} = 1, I_{2i} = 1) &= \frac{N}{(N-1)}\left(\frac{N}{n_1} - 1\right)(y_i - \bar{Y}) \\
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- Poisson sampling/Poisson sampling:

$$\begin{aligned} B_i^{DE}(I_{1i}=1, I_{2i}=1) &= \left(\frac{1}{\pi_{1i}}-1\right)y_i + \frac{1}{\pi_{1i}}\left(\frac{1}{\pi_{2i}}-1\right)y_i \\ &= \left(\frac{1}{\pi_i^*}-1\right)y_i \end{aligned}$$

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A robust version of the double expansion estimator

- Following Beaumont, Haziza and Ruiz-Gazen (2011), we obtain

$$\hat{Y}_{DE}^{R} = \hat{Y}_{DE} - \sum_{i \in s_2} \hat{B}_i^{DE}(I_{1i} = 1, I_{2i} = 1) + \sum_{i \in s_2} \psi \left\{ \hat{B}_i^{DE}(I_{1i} = 1, I_{2i} = 1) \right\}$$

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$$\psi(t) = \begin{cases} c & \text{if } t > c \\ t & \text{if } |t| \leq c \\ -c & \text{if } t < -c \end{cases}$$

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- Special case: single-phase sampling; i.e., $I_{2i} = 1$ for all $i \Rightarrow \hat{Y}_{DE}^{R}$ reduces to the robust estimator proposed by Beaumont, Haziza and Ruiz-Gazen (2011).

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- Influence of a responding unit:

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- Parametric nonresponse model: $\pi_{2i} = m(\mathbf{x}_i, \boldsymbol{\alpha})$, where
  - m(.) is a known function
  - $\mathbf{x}_i$ is a vector of auxiliary variables available for all the sampled units (respondents and nonrespondents)
  - $\boldsymbol{\alpha}$ is a vector of unknown parameters

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- Parametric nonresponse model: $\pi_{2i} = m\left(\mathbf{x}_i, \boldsymbol{\alpha}\right)$, where
  - m(.) is a known function
  - $\mathbf{x}_i$ is a vector of auxiliary variables available for all the sampled units (respondents and nonrespondents)
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- Robust version of $\hat{Y}_{PSA}$

$$\begin{aligned} \hat{Y}^R_{PSA} &= \hat{Y}_{PSA} - \sum_{i \in s_2} \hat{B}_i^{PSA}(I_{1i} = 1, I_{2i} = 1) \\ &+ \sum_{i \in s_2} \psi \left\{ \hat{B}_i^{PSA}(I_{1i} = 1, I_{2i} = 1) \right\} \end{aligned}$$

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- Generate nonresponse: Bernoulli trials with probability π_{2i}, where

$$\pi_{2i} = \frac{1}{\exp(\alpha_0 + \alpha_1 x_i)}$$

- Global response rate: 70%

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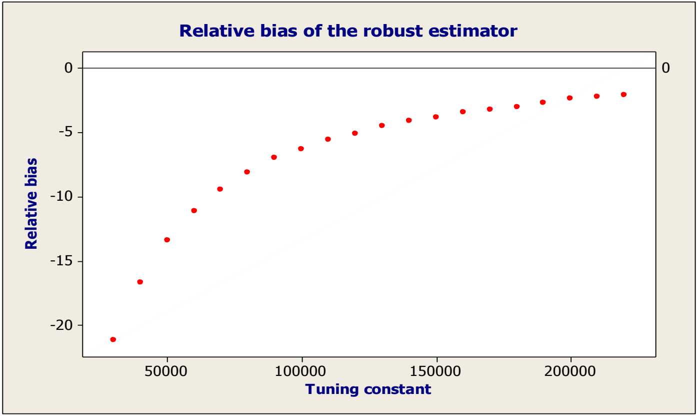
    - Relative Efficiency with respect to the nonrobust estimator:

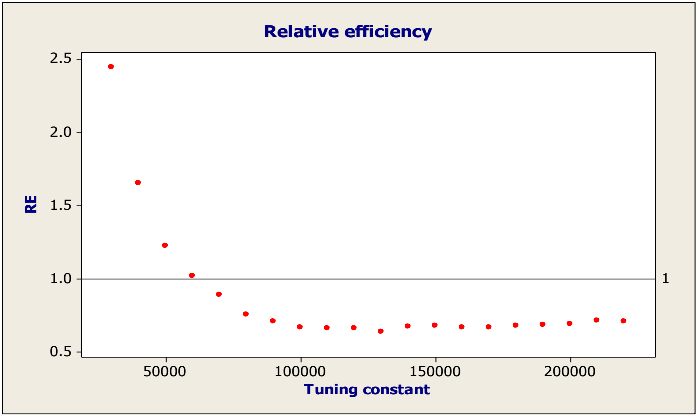
$$RE(\hat{Y}^{R}_{PSA}) = \frac{MSE(\hat{Y}^{R}_{PSA})}{MSE(\hat{Y}_{PSA})}$$

- Note: $\hat{Y}_{PSA}$ has negligible bias

Relative bias of the robust estimator (5% contamination)



Relative efficiency with respect to the nonrobust estimator (5% contamination)



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- Results can be extended to the case of calibration estimators $\Rightarrow$ important in the unit nonresponse context since weight adjustment procedures by the inverse of the estimated response probabilities are generally followed by some form of calibration
- Requires further investigations:
  - Choice of the tuning constant
  - MSE estimation: reverse framework for variance estimation?