A Dynamic Model of Sponsored Search

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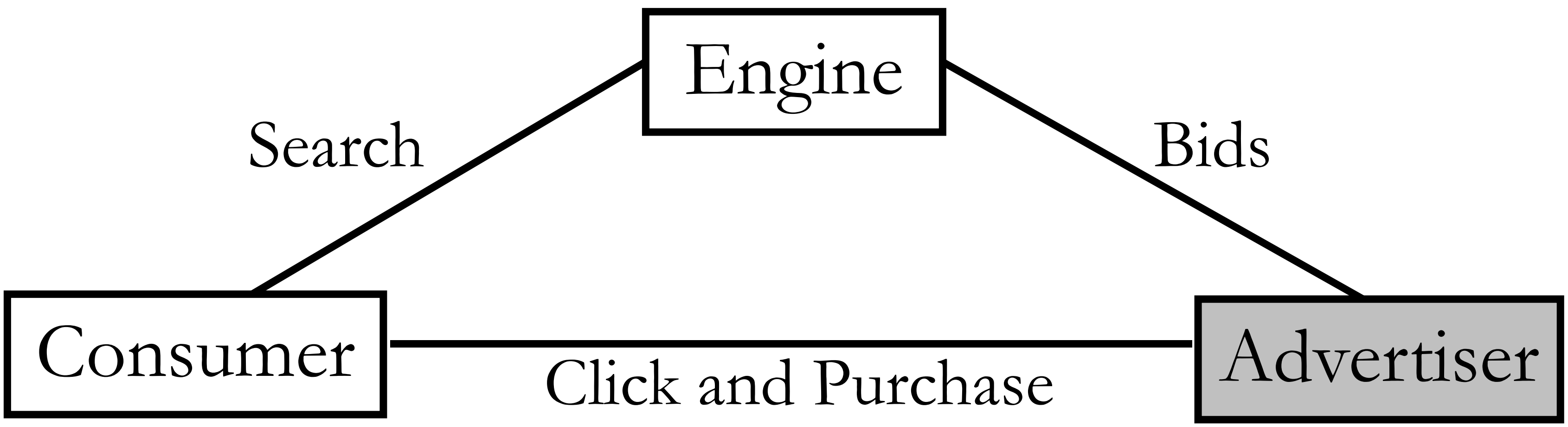
Research Goals

- Framework to assess effect of search engine strategy on its revenues
  - Bids and clicks jointly determine search revenues
  - Using click histories, assess how consumers search
  - Using bid histories, impute advertisers' expected profit for a key word advertisement

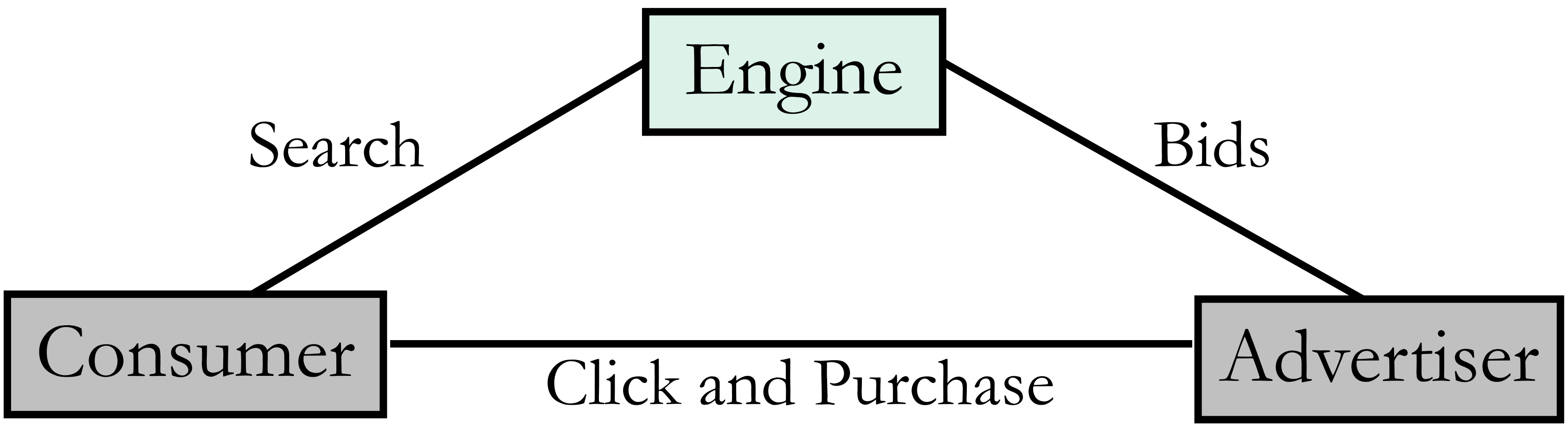
Research Implications

- Marketing to searcher/consumer
  - Webpage design: sort, filter, rank, etc.
- Marketing to advertiser
  - Market intelligence: search engine knows person's ad clicks, advertisers do not
  - Targeting: bid by key word and segment
  - Auction mechanism: first or second price

Prior Literature



This Research



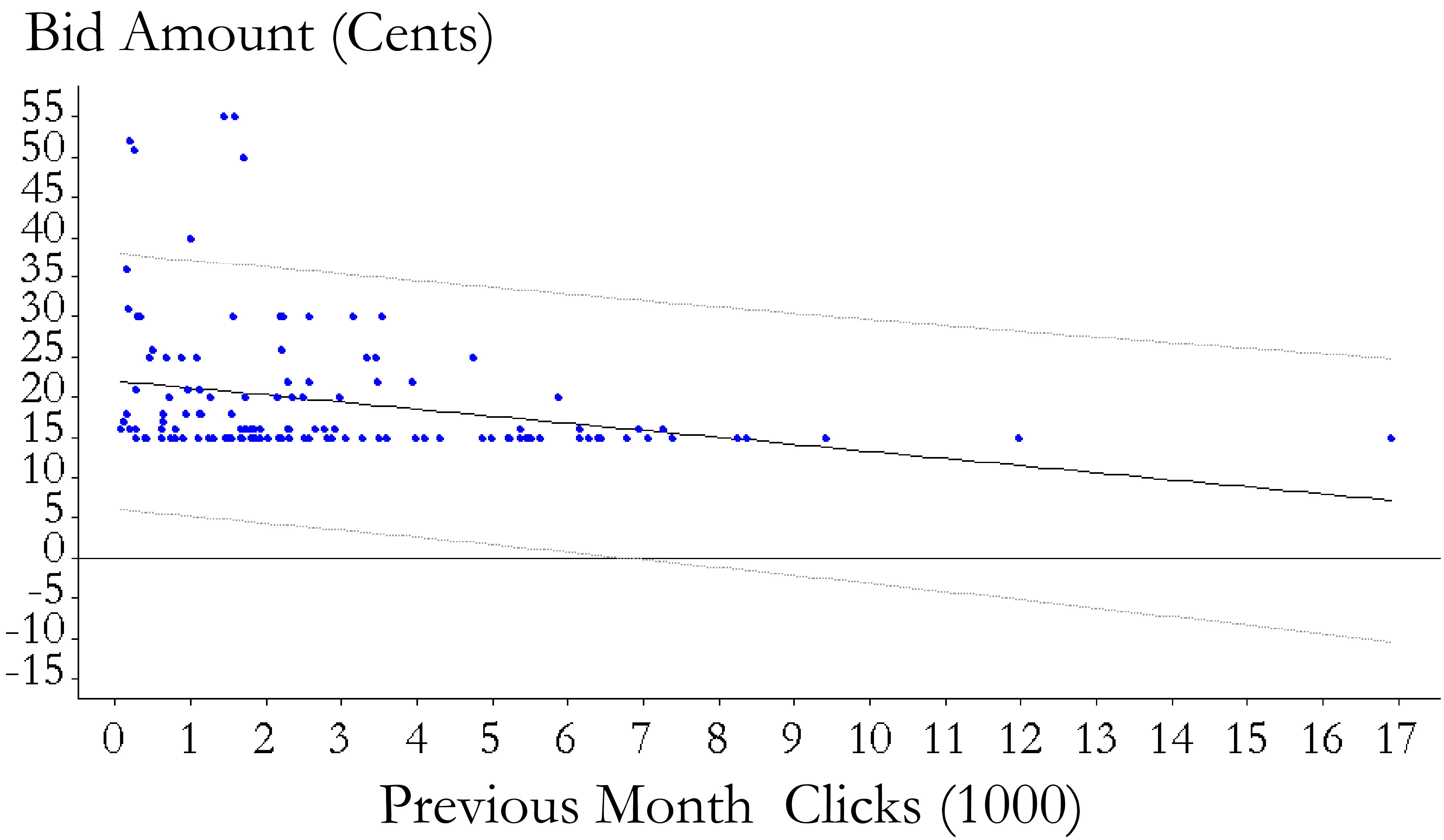
Data

- Advertiser
  - 32 months for 21 bidders
  - Bid level, time of bid, slot positions, total clicks
  - Product attributes (downloadable software)
- Consumer
  - 4 months
  - Browsing and click history

Bidding Rules

- Monthly auction, simultaneous bids
- Top 5 win sponsored slots in current period
- Winners pay their bids for each click
- Bidders ranked by bid*last period clicks
  - Hence, bidding problem is dynamic

Advertiser Behavior



Advertiser Data

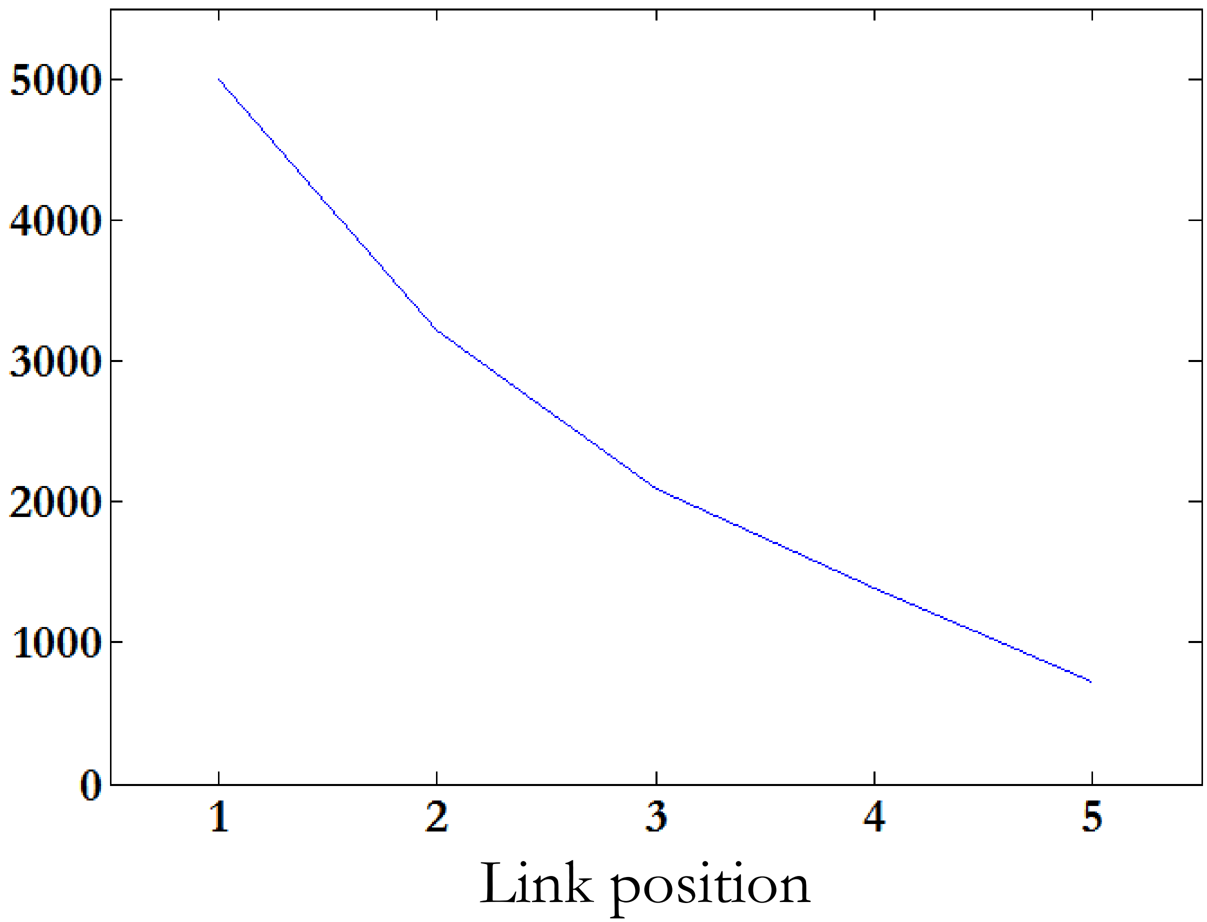
	Mean	Std. Dev.	Minimum	Maximum
Non-zero Bids (¢)	19.55	8.32	15	55
Non-zero Bids/Bidder	21.78	10.46	1	30
All Bids (¢)	8.14	11.04	0	55
Bids/Bidder	23.13	9.68	1	32

Product Attributes Data

	Mean	Std. Dev.	Minimum	Maximum
All Products				
Non-trial Version Price $	16.65	20.43	0	150
Expert Rating (if rated)	3.87	0.81	2	5
Average Consumer Rating (if rated)	3.89	1.31	1	5
Months Lapse Since Last Update	15.31	9.88	1	31
Compatibility Index	3.29	1.47	0	5
Bidders' Products				
Non-trial Version Price $	21.97	15.87	0	39.95
Expert Rating (if rated)	4	0.50	3	5
Average Consumer Rating (if rated)	4.06	0.91	2.5	5
Months Lapse Since Last Update	2.38	0.66	1	3
Compatibility Index	3.51	1.51	0	5

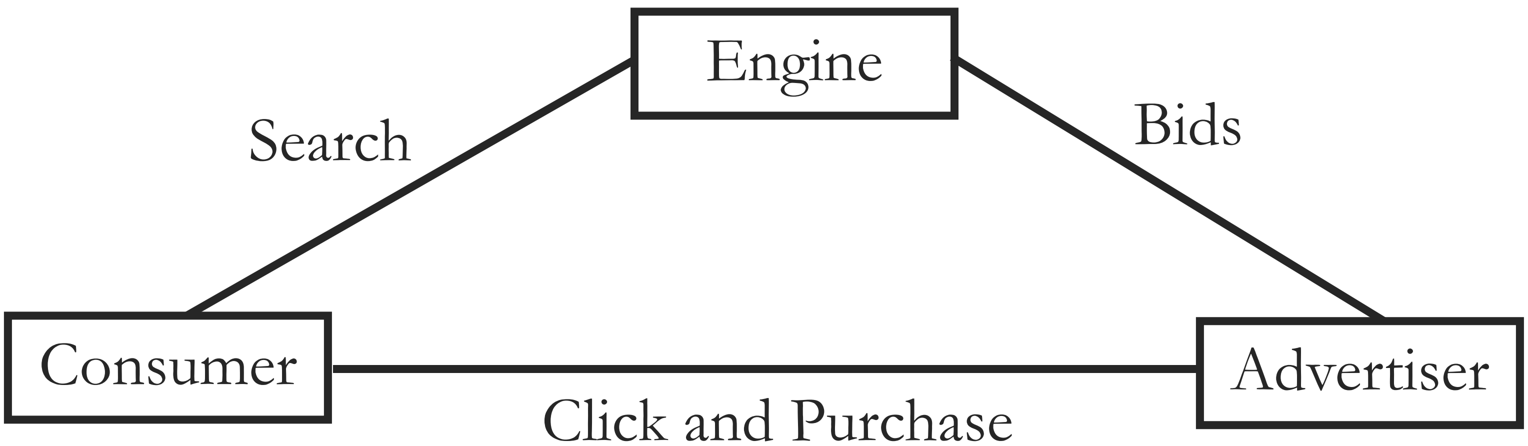
Consumer Click Data

Mean clicks per product per month



Model Overview

Stage 1: Search Engine



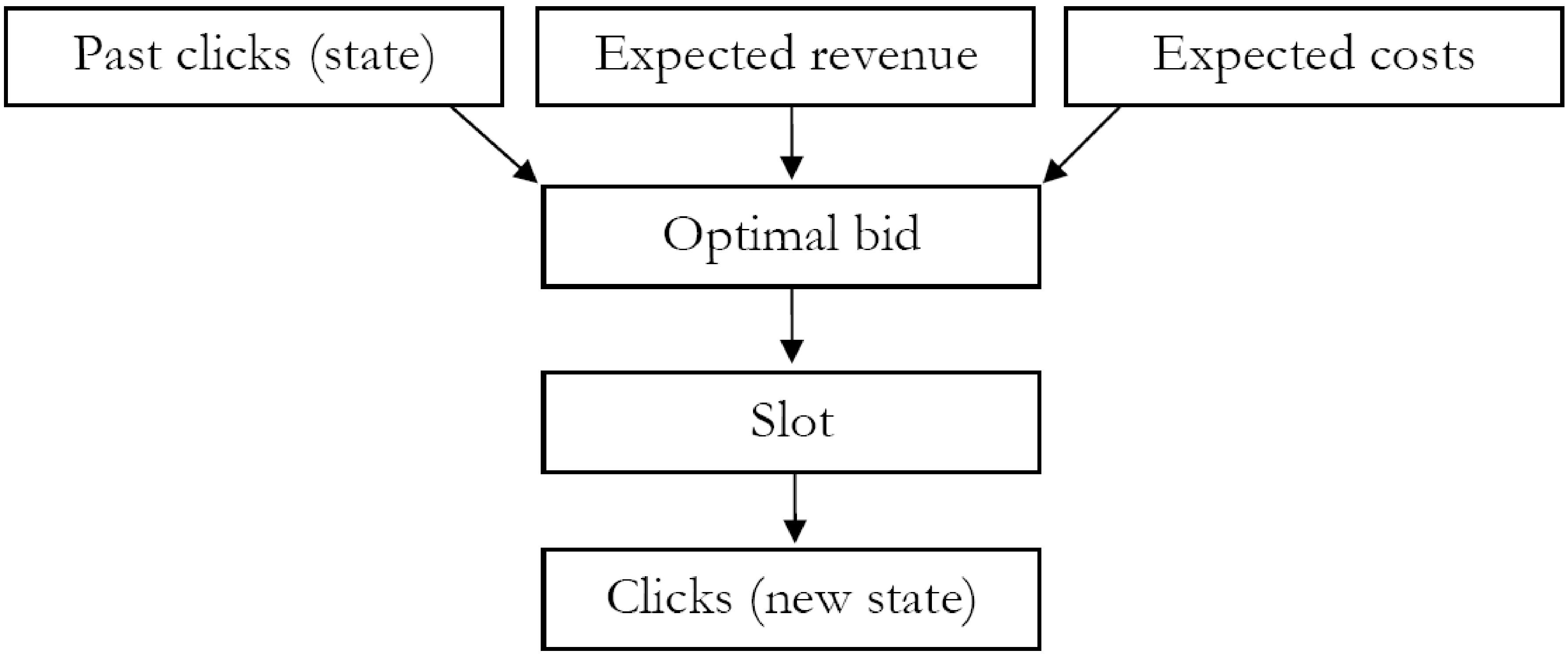
Stage 3: Consumers

Click to max utility

Stage 2: Advertisers

Bid to max discounted profits

Stage 2: Advertiser Model



Advertiser Revenues and Costs

- Monthly revenues: $v_j^t d_j^t$
  - $v_j^t = X_j^t \theta + f_j + r_j^t$ is click value to infer
  - d_j^t are clicks from consumer model
  - *j* is firm, *t* is month and *r* are random terms
- Monthly costs if win: $b_j^t d_j^t$
  - b_j^t is the current period's bid (decision variable)
  - No costs per click if lose

Advertiser Period Profits

- Monthly profit = paid prof. + organic prof.

$$
\begin{aligned}
&\mathbf{E}\pi_j\left(\mathbf{b}^t, \mathbf{s}^t, \mathbf{X}^t, r_j^t; \theta, f_j\right) \\
= &\sum_{k=1}^{K} \Pr\left(k | b_j^t, \mathbf{b}_{-j}^t, \mathbf{s}^t, \mathbf{X}^t\right) \cdot \left(v_j^t - b_j^t\right) \cdot d(k, X_j^t; \Omega_c) \\
&+ \sum_{k=K+1}^{\overline{N}} \Pr\left(k | b_j^t, \mathbf{b}_{-j}^t, \mathbf{s}^t, \mathbf{X}^t\right) \cdot v_j^t \cdot d(k, X_j^t; \Omega_c)
\end{aligned}
$$

probability getting slot k

value per click

clicks (from consumer model)

back

Advertiser Total Profit

- Total profits are discounted sum of period profits:

value function

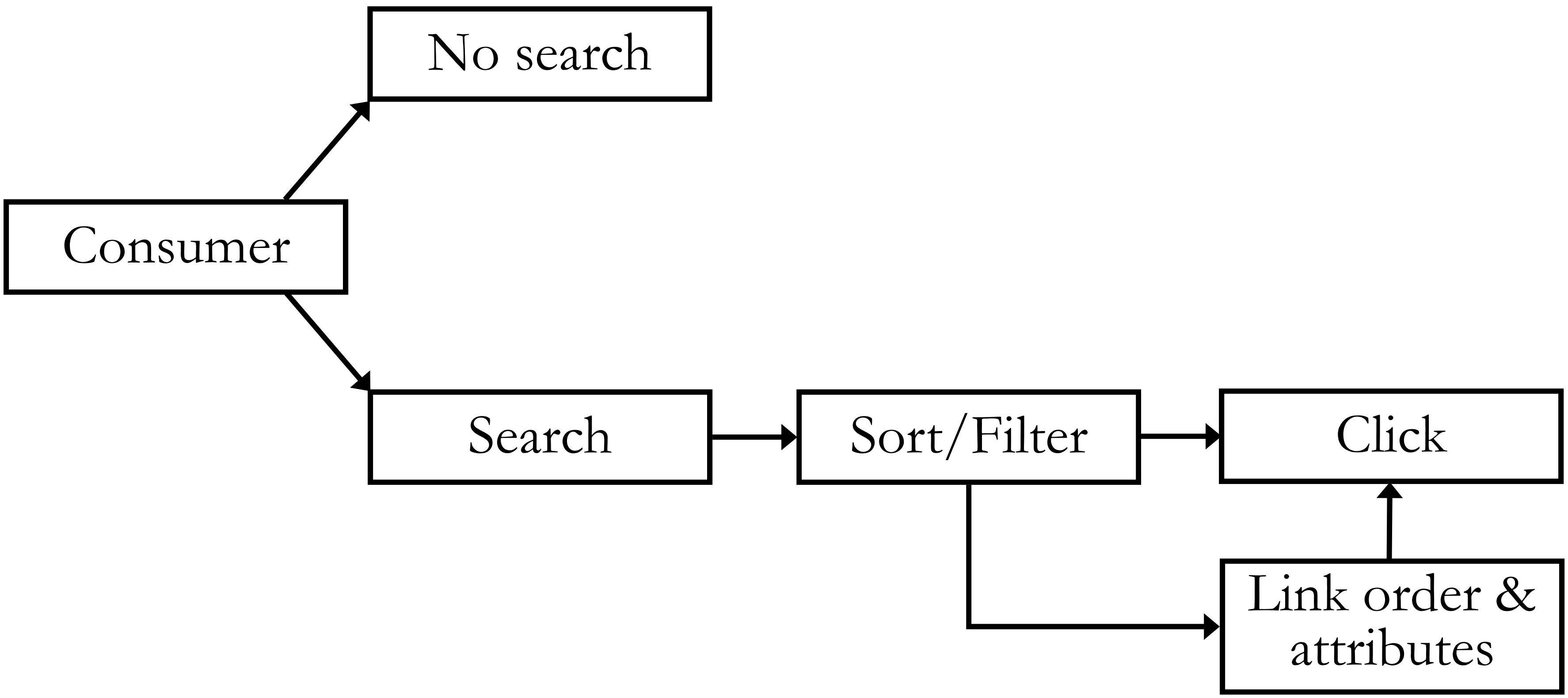
$$V(d_j^t, b_j^t, v_j^t | \mathbf{s}^t, \mathbf{X}^t)$$
$$= E[\textstyle\sum_{\tau=t}^{\infty} \rho^{\tau-t} \pi(d_j^\tau, b_j^\tau, v_j^\tau | \mathbf{s}^\tau, \mathbf{X}^\tau)]$$

discount rate

advertiser period profits

- Advertiser chooses bid to max total profit

Stage 3: Consumer Model



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Click Likelihood

- Click likelihood is product of three decisions
  - p(click|strategy,search)
  - p(strategy|search)
  - p(search)
- Heterogeneity accommodated via a latent class structure

detail back

Stage 1: Search Engine Strategy

- Consumer
  - Webpage design detail
- Advertiser
  - Market intelligence detail
  - Targeting detail
  - Auction mechanism detail

Estimation

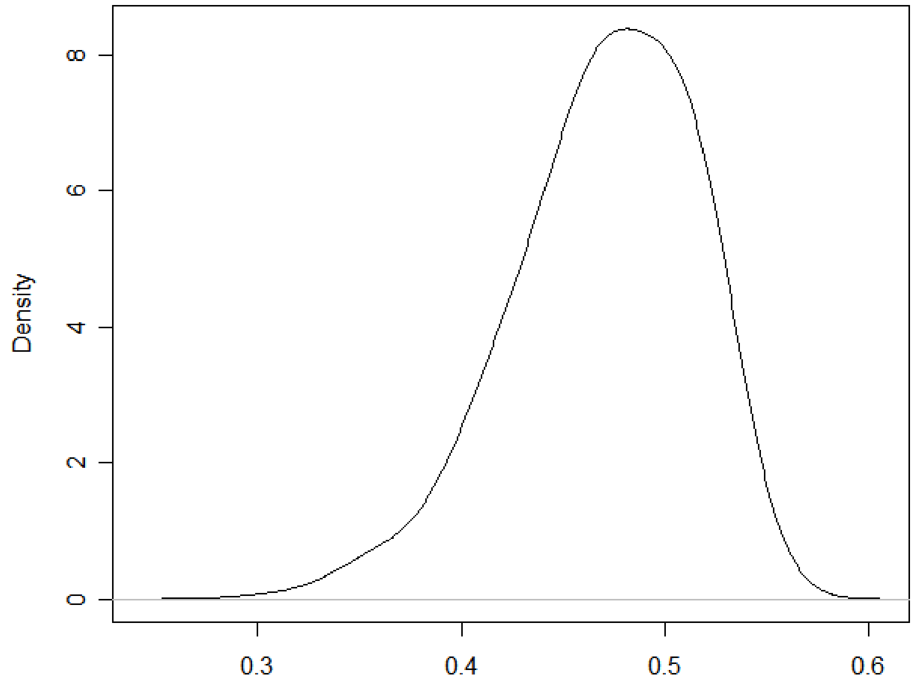
- Two-step estimation (Bajari et. al. 2007):
  - Profit function requires $b_j^t(\mathbf{s}^t, \mathbf{X}^t), d_j^t(\mathbf{X}^t), v_j^t(\mathbf{X}^t)$
  - Step 1: $b_j^t(\mathbf{s}^t, \mathbf{X}^t), d_j^t(\mathbf{X}^t)$
    - Advertiser behavior: tobit yields bids $b_j^t(\mathbf{s}^t, \mathbf{X}^t)$ as function of states and attributes
    - Consumer behavior: choice model yields clicks $d_j^t(\mathbf{X}^t)$ as function of attributes
    - Substitute $b_j^t(\mathbf{s}^t, \mathbf{X}^t)$ and $d_j^t(\mathbf{X}^t)$ into profit function and obtain $v_j^t(\mathbf{X}^t)$ in Step 2

Estimation

- Two-step estimation (continued):
  - Step 2: obtain advertiser value parameters, v_j^t
    - compute profit recursion as function of v_j^t
    - choose v_j^t to yield highest V such that no deviation from optimal bids yields higher profit

Bayesian Estimation

Expert Rating Parameter Posterior
Advertiser Value Model



K.S. test distance=0.99, $p<0.01$

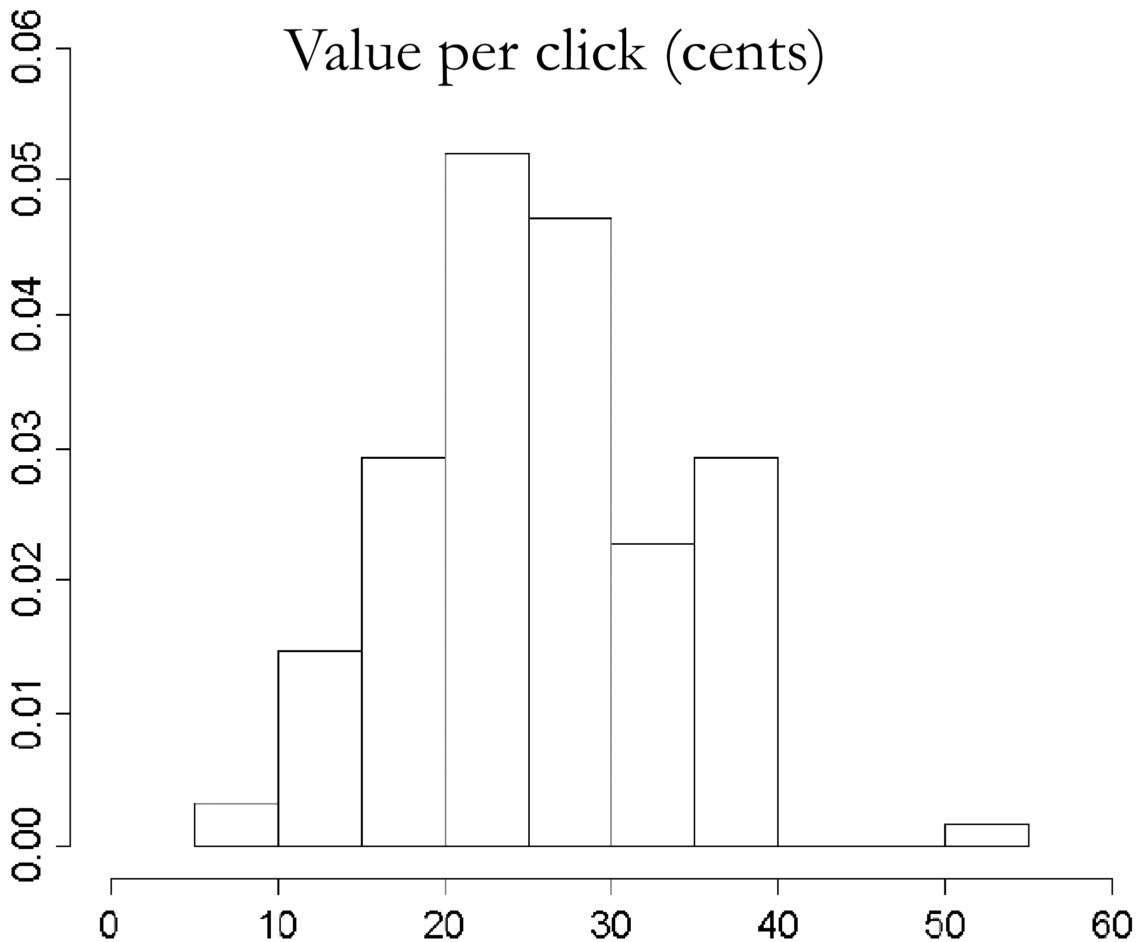
First Step: Advertiser Bidding Policy

	Median	95% Interval
φ		
Lagged Downloads$_{jt}/10^3$	-0.12^*	$(-0.16, -0.09)$
Total Lagged Downloads$_t/10^3$	0.04^*	$(0.01, 0.08)$
Lagged Number of Bidders$_t$	0.04^*	$(0.01, 0.06)$
Lapse Since Last Update$_{jt}$	-0.31^*	$(-0.70, -0.02)$
Non-trial Version Price$_{jt}$	0.36^*	$(0.33, 0.39)$
Expert Ratings$_{jt}$	0.47	$(-0.26, 1.23)$
Consumer Ratings$_{jt}$	0.81^*	$(0.10, 1.50)$
Compatibility Index$_{jt}$	-0.91^*	$(-1.74, -0.20)$
Lagged MP3 Player Price$_t$	0.03^*	$(0.02, 0.03)$
τ	8.14^*	$(6.78, 11.37)$
Log Marginal Likelihood	-1123.59	

First Step: Consumer Model

	Segment 1 (89.5%) (Infrequent searcher)	Segment 2 (10.5%) (Frequent searcher)
	Median (95%Interval)	Median (95%Interval)
β^g (utility parameters)		
Constant	-0.09 $(-0.11, 0.001)$	0.35 $(0.31, 0.38)$
Slot Rank	-0.08 $(-0.06, -0.09)$	-0.51 $(-0.52, -0.50)$
Non-trial Version Price	0.03 $(0.03, 0.04)$	-0.04 $(-0.04, -0.03)$
Expert Ratings	0.16 $(0.15, 0.17)$	0.06 $(0.06, 0.07)$
Consumer Ratings	0.11 $(0.11, 0.12)$	0.03 $(0.03, 0.05)$
Compatibility Index	-0.08 $(-0.09, -0.07)$	0.16 $(0.16, 0.17)$
Total Click Percentage	0.01 $(-0.02, 0.05)$	0.09 $(0.08, 0.10)$
δ^g (sorting/filtering scaling)	1.52 $(1.48, 1.55)$	1.87 $(1.78, 1.99)$
λ^g (search probability)		
λ_0^g (base)	-10.22 $(-10.75, -9.60)$	-0.78 $(-1.21, -0.54)$
λ_1^g (1-correlation)	0.02 $(0.01, 0.02)$	0.03 $(0.01, 0.04)$

Second Step: Advertiser Value



Second Step: Advertiser Value

- Mean value per click is 27 cents
  - Implied click/sale conversion rate is \$0.27/\$22.00 = 1.2%
  - Industry Average 1-2% (Gamedaily.com)
- Attributes that enhance product quality correlate with higher values per click

Advertiser Policy Simulations

- Webpage design: remove sort, filter, etc.
  - Search Revenue -3.7% (-6.4% cons.; +2.7% advt.)
  - Consumer welfare decreases 5.6%
  - Advertiser profits increase 4.1%

Advertiser Policy Simulations

- Targeting: bid by key word and segment
  - Revenue +1.4% (+2.2% cons.; -0.8% advt.)
  - Advertiser revenue increases 11%
  - Consumer welfare increases 2.9%
- Welfare gains are positive for all agents

Advertiser Policy Simulations

- Market intelligence: search engine knows person's ad clicks, advertisers do not
  - No change in profits or welfare
  - Need to couple with targeting
- Auction mechanism: first to second price
  - Bids/Values 72% to 98% (truth telling)
  - However, no effect on search profits (0.01%)

Contribution

- Substantive Contribution
  - Findings (e.g., valuations)
  - Prescriptions (e.g. consumer and advertiser)
- Methodological Contribution
  - Bayesian 2 step estimation
- Theoretical Contribution
  - Dynamic 2 sided network for key words

Future Potential Directions

- More advertiser problems
  - Choice of key words
  - Multi-word bidding
  - Other mechanism considerations
    - Continuous/discrete time bidding
    - Bidding caps

Appendix

Policy Simulations

- Two-step estimates reflect old policy rules
    - Now need to solve dynamic program (DP)
- Approximate dynamic program (Judd, 1998)
    - Estimate bidding rule in lieu of bidding path
- Reduce state space
    - Own and total clicks instead of all firms' clicks
    - Own clicks only (oblivious equilibrium (Weintraub, Benkard and van Roy, 2008))
- Other solutions detail

Consumer Click

- A consumer clicks if expected utility from doing so is greater than zero
  - Link position
  - Product attributes
- Leads to probit model of clicks

detail

Consumer Sort and/or Filter

- Consumers sort and/or filter if their expected utility from doing so increases
  - For example, does sorting by price increase the expected utility of the search alternatives?
    - Yes, if one is price sensitive
    - No, if i) one cares more for quality, ii) initial query lists high quality goods first, and iii) low price alternatives are also lower quality ones
  - Leads to logit choice model for search strategy

detail

Consumer Search Decision

- Consumers choose to search if the expected maximum utility across search strategies is greater than zero
- Leads to nested logit model of search decision
  - Lower nest is search strategy
  - Upper level is search/no search

detail

Consumer Clicks

- Click if utility $u_{ijt}^{g\kappa} > 0$
  - $u_{ijt}^{g\kappa} = \widetilde{\alpha}_j^g + x_{jt}^{\kappa}\widetilde{\beta}^g + \widetilde{\varepsilon}_{ijt}^{g\kappa}$
    - Utility, $u_{ijt}^{g\kappa}$ for consumer i at time t
    - Attributes, x
    - Parameters, α and β
    - Segment, g (latent class model)
    - Sort/filter option, κ
- Yields probit click model

back

Consumer Sort and Filter

- Expected utility for sort/filter strategy κ

$$U_{it}^{g\kappa} = \sum_j \underbrace{E_\varepsilon(u_{ijt}^{g\kappa} | u_{ijt}^{g\kappa} \geq 0)}_{\text{utility of prod. } j \text{ given strategy } \kappa} \underbrace{\Pr(u_{ijt}^{g\kappa} \geq 0)}_{\text{likelihood click } j \text{ with strategy } \kappa}$$

- Probability choose sort/filter strategy κ

$$\Pr(\kappa)_{it}^{g} = \frac{\exp(U_{it}^{g\kappa})}{\sum_{\kappa'=0}^{3} \exp(U_{it}^{g\kappa'})}$$

back

Consumer Search Decision

- Consumer searches key word if expected utility across search strategies is positive
  - Nested logit where inclusive value is:

$$IV_{it}^{g} = \log[\sum_{\kappa} \exp(U_{it}^{g\kappa})]$$

  - Leading to search probability:

$$\Pr(search_i^g) = \frac{\exp(\lambda_0^g + \lambda_1^g IV_{it}^g)}{1 + \exp(\lambda_0^g + \lambda_1^g IV_{it}^g)}$$

back

Click Likelihood

probability of click given sort/filter option κ (probit) * probability of sort/filter strategy given search (logit) * probability of search (nested logit)

$$P_{ijt} = \int_{Demo_{it}} \sum_g \sum_\kappa \left[\Phi(\overline{u}_{ijt}^{g\kappa}) \frac{\exp(U_{it}^{g\kappa})}{\sum\limits_{\kappa'=0}^{3} \exp(U_{it}^{g\kappa'})}\right] \Pr(search_{it}^{g}) pg_{it}^{g} d\mathcal{D}(Demo_{it})$$

average over segments, strategies and demographics (not known to advertiser)

back

Consumer Model: Heterogeneity

- Targeting and value of information
- Latent class model where:

$$pg_{it}^{g} = \frac{\exp(\gamma_0^g + Demo'_{it}\gamma^g)}{\sum_{g'=1}^{G} \exp(\gamma_0^{g'} + Demo'_{it}\gamma^{g'})}$$

Probability of being in segment *g*

Demographics and browsing history

back

Consumer Model: Scaling Parameter

$$u_{ijt}^{g\kappa} = \widetilde{\alpha}_j^g + \sum\nolimits_a x_{jat}^{\kappa} \widetilde{\beta}_a^g + \widetilde{\varepsilon}_{ijt}^{g\kappa}$$

$$= \delta^g \big(\underbrace{\alpha_j^g + \sum\nolimits_a x_{jat}^{\kappa} \beta_a^g}_{\overline{u}_{ijt}^{g\kappa}} + \varepsilon_{ijt}^{g\kappa} \big)$$

Std. dev. of $\widetilde{\varepsilon}_{ijt}^{g\kappa}$

$$U_{it}^{g\kappa} = \sum_j E_{\varepsilon}(u_{ijt}^{g\kappa} | u_{ijt}^{g\kappa} \geq 0) \cdot \Pr(u_{ijt}^{g\kappa} \geq 0)$$

$$= \delta^g \sum_j \left(\overline{u}_{ijt}^{g\kappa} + \frac{\phi(\overline{u}_{ijt}^{g\kappa})}{\Phi(\overline{u}_{ijt}^{g\kappa})} \right) \cdot \Phi(\overline{u}_{ijt}^{g\kappa})$$

back

DP in Policy Simulations

- Other solutions
  - Local perturbation starting around the original equilibrium
  - Continuous-time DP (Doraszelski and Judd, 2008; Nekipelov, 2008)
  - Sequential state-to-state transition (Doraszelski and Judd, 2007)
  - Static game

back

Parametric vs. Nonparametric

- The objective of first step: approximate the equilibrium play
- The ideal solution: non-parametric first step
  - Data requirement
  - Still needs arbitrary assumptions (e.g., bin width in Kernel estimator)
- Flexible parametric specifications
  - Choose the one with the best fit

back

Advertiser Heterogeneity

- Panel data help to identify the unobserved heterogeneity (e.g., BBL; Honde and Imai, 2006)
- Other methods for identifying heterogeneity include Kasahara and Shimotsu (2008), Hu and Shum (2008)

back

Segment Specific Values

- Monthly prof.=paid prof. from each seg. +organic prof. from each seg.

$$\mathbf{E}\pi_j\left(\mathbf{b}^t, \mathbf{s}^t, \mathbf{X}^t, r_j^t; \theta, f_j\right)$$

$$= \sum_{g=1}^{G}\left[\sum_{k=1}^{K}\Pr\left(k|b_j^t, \mathbf{b}_{-j}^t, \mathbf{s}^t, \mathbf{X}^t\right)\cdot\left(v_{jg}^t - b_j^t\right)\cdot d_g(k, X_j^t; \Omega_c)\right.$$

$$\left. + \sum_{k=K+1}^{\overline{N}}\Pr\left(k|b_j^t, \mathbf{b}_{-j}^t, \mathbf{s}^t, \mathbf{X}^t\right)\cdot \underbrace{v_{jg}^t}_{\text{Segment specific value}}\cdot \underbrace{d_g(k, X_j^t; \Omega_c)}_{\text{Segment specific clicks}}\right]$$

back

Slot Specific Values (Conversion)

- Monthly prof.=paid prof. from each slot +organic prof. from each slot

$$\begin{aligned}&\mathbf{E}\pi_j\left(\mathbf{b}^t,\mathbf{s}^t,\mathbf{X}^t,r_j^t;\theta,f_j\right)\\=&\sum_{k=1}^{K}\Pr\left(k|b_j^t,\mathbf{b}_{-j}^t,\mathbf{s}^t,\mathbf{X}^t\right)\cdot\left(v_{jk}^t-b_j^t\right)\cdot d(k,X_j^t;\Omega_c)\\&+\sum_{k=K+1}^{\overline{N}}\Pr\left(k|b_j^t,\mathbf{b}_{-j}^t,\mathbf{s}^t,\mathbf{X}^t\right)\cdot v_{jk}^t\cdot d(k,X_j^t;\Omega_c)\end{aligned}$$

Slot specific value

Slot specific clicks

back

Second Step: Advertiser Model

Table 8: Alternative Models

Model	Log Marginal Likelihood
Base Model	-1651.3
Base Model Without Advertiser Dynamics	-1701.3
Base Model With Heterogeneous Customer Valuations	-1645.1